

# MODELLING OF INDIRECT VISIBILITY

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## ABSTRACT

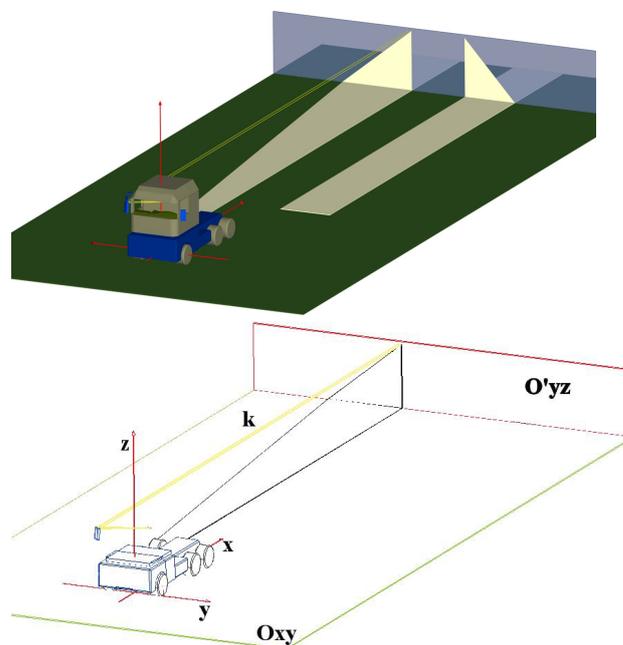
It is necessary to retrofit the vehicles and road infrastructure with the devices of indirect visibility. For the purpose of the tests, a mathematical model of the tests' subject was created, which was used to write the analytical computer program for the PC class computer. The mathematical model of the visual image transfer from the vehicle's surroundings transmitted by the mirrors encompasses: driver's eyes represented by the ocular points placed in the three-dimensional, rectangular left-handed, Cartesian system, the vehicle's block, described in this system by the clear and opaque surfaces; mirrors of the known features, mounted on the vehicle's block; the space surrounding the vehicle placed on the horizontal surface. This mathematical model was used to create the calculation program in the C++ programming language and using the Open GL library, working under the Microsoft Windows operating system. In the specific range of values it is possible to increase or decrease the size of the presented object – it helps to see the details, to change the direction of the observations. The operating program relies on an input of such quantities as width of the vehicle, location and extent of the transparent elements. The location of ocular points, and regions which driver should observe, in respect to the vehicle are also set. The results of the program calculations are presented graphically as the virtual picture of reality on the computer monitor. Apart from that, the program presents in the tabular form, values of coordinates of the points on the planes  $Oxy$  and  $O'yz$ , calculated for the given step, and for accepted parameters. Additionally there is a possibility to draw the regions covered by the eyesight. The created model of the visual transfer from the vehicle's surroundings, transmitted by the mirror enables to conduct the virtual tests of the real objects. The necessary data such as: position of ocular points in the real vehicle, the location of the mirrors and their features, width of the vehicle must be measured in the vehicle and introduced into the program.

**Key words:** mathematical model, indirect visibility, safety.

## INTRODUCTION

There is the need to retrofit the vehicles and road infrastructure with the devices of indirect visibility. For the purpose of tests there was a mathematical model of the tests' subject created, which was used to write the analytical computer program for the PC class computer [1, 2]. The mathematical model of the visual image transfer from the vehicle's surroundings transmitted by the mirrors projects:

- driver's eyes by the ocular points placed in the three – dimensional space – Cartesian system, rectangular left-handed,
- the vehicle's block, described in this space by the clear and opaque surfaces,
- mirrors of the known features, mounted on the vehicle's block,
- the space surrounding the vehicle placed on the horizontal surface (Fig. 1.).



**Fig. 1. Model of the real object of the tests and the diagram of the arrangement in the coordinates system**

The front of the vehicle is turned in the opposite direction to that of the  $Ox$  axle. The coordinates

system (with respect to the vehicle) is placed in the following way:

- the vertical surface  $Oxz$  runs through the longitudinal axle of the vehicle,
- the vertical surface  $Oyz$  is perpendicular to the surface  $Oxz$  which runs through the eyes' points,
- the horizontal surface  $Oxy$  lies on the road surface, which the vehicle stands on.

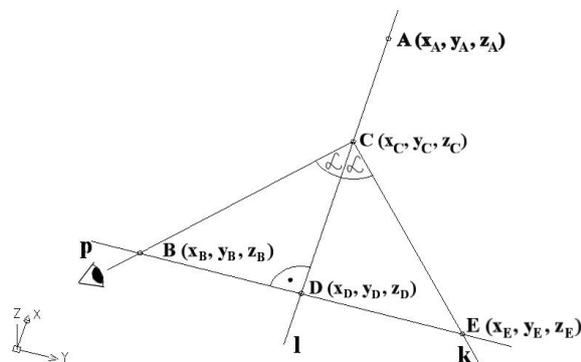
On the horizontal surface  $Oxy$  there are areas of the vehicle's surroundings projected, which, for example, in accordance with the current regulations, the driver should have the possibility to observe. The line of the horizon is projected on the surface  $O'yz$  which is parallel to the surface  $Oyz$ .

The surface  $O'yz$  is at the rear behind the vehicle in the distance of the given value from the surface  $Oyz$ , Fig. 1.

## MATHEMATICAL MODEL

Having given points  $A$ ,  $B$ ,  $C$  in the space, which project the following:

- $A$  – the center of the sphere (sphere on which there is the surface of the mirror),
- $B$  – one of the ocular points,
- $C$  – point on the mirror, at which the observer's eyesight is directed; we look for the equations of the straight line  $CE$ . On the drawing no 2 there is a position of the mentioned points illustrated.



**Fig. 2. The model of the elementary reflection problem in the mirror for the specular reflector placed on the vehicle**

Coordinates of point  $D$  on line  $I$  were determined by drawing line  $p$  through point  $B$ , which is perpendicular to line  $I$ . Because the vectors of line  $I$  and line  $p$  are not zero, in accordance with the properties of the scalar product, the vectors of these lines, it transpires that when the scalar product of vectors produces the zero result, then the lines  $I$  and  $p$  are perpendicular to each other.

The common point (the point of crossing) of lines  $I$  and  $p$  was marked as  $D$ . The common point of lines  $I$  and  $k$  was marked as  $E$ . The point  $D(x_D, y_D, z_D)$  found, enables us to determine the coordinates of

the point  $E(x_E, y_E, z_E)$  as the symmetrical one to the point  $B$  against the line  $I$ . Coordinates of the point  $E$  because of symmetry against line  $I$  are described by the following equations (1):

$$\begin{aligned} x_E &= 2 \cdot x_D - x_B \\ y_E &= 2 \cdot y_D - y_B \\ z_E &= 2 \cdot z_D - z_B \end{aligned} \quad (1).$$

Thus points  $C$  and  $E$  determine the line  $k$  in the space. Knowing the equation of line  $k$  we can calculate coordinates of the point of the penetration of planes  $Oxy$  and  $O'yz$  by this line.

Points of the penetration of these planes by line  $k$ , were reflected on the whole surface of the mirror, and provide the ability to determine the place and the extent of the region observed by one eye for the given location and setting of the mirror. Similarly we should act to determine the region observed by the second eye. Changing the values of point  $A$  coordinates in such a way that it moves on the surface of the sphere whose radius is determined by the interval  $AC$  and its centre, by point  $C$ , we set the mirror in such a way to direct the observer's eyesight at the area around the vehicle, which the observer wants to see. By analyzing the area around the vehicle, the area on the planes  $Oxy$  and  $O'yz$  for example, which is compatible with the current rules or with the expectations, we can determine:

- if for the given mirror and its placement on the vehicle the driver can take in a view of the required area,
- what conditions must be fulfilled (height, width, radius of curvature of the mirrors, parameters of its placement on the vehicle, settings) to enable the driver to take in a view the required area for the specific vehicle,
- the limitations of the use of mirrors for the indirect visibility of the regions of the vehicle's surroundings for the specific category of the vehicle and the constructional solutions,
- sensitiveness of the tests object (which is the function of many variables) to the particular variables.

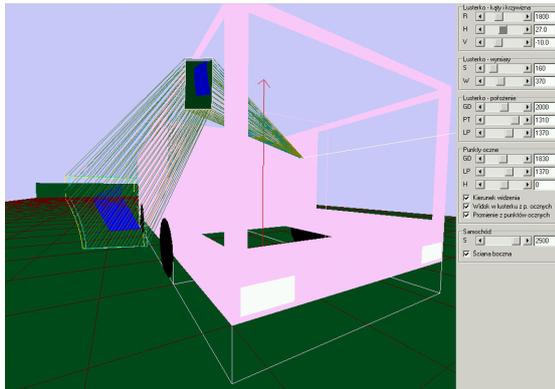
This mathematical model was used to create the calculation program in the C++ programming language and using the Open GL library, working under the Microsoft Windows operating system. In the specific range of values it is possible:

- to increase or decrease presented object – it helps to watch details,
- to change the direction of the observations.

The operating program relies on an input of such quantities as width of the vehicle, location and extent of the transparent elements. In addition, to setting the location with respect to the vehicle, it is also possible to include the following factors:

- ocular points,
- regions which driver should observe,

- mirrors which are considered (its width, height and radius of curvature).



**Fig. 3. The example of the picture, generated by the program, received in the monitor**

Next, while changing the angles of the mirror adjustment vertically and horizontally we check if the region covered by the driver's eyesight through the mirrors will cover completely the region determined by the regulations or given as the result of the other limitations.



**Fig. 4. View the real measuring stand with the manikin and the halogen head**

The results of the program calculations are presented graphically as the virtual picture of reality on the computer monitor. Apart from that, the program presents values of coordinates of the points on the planes  $Oxy$  and  $O'yz$ , calculated for the given step, and accepted parameters in the tabular form. Additionally there is a possibility to draw the regions covered by eyesight.

The created model of the visual transfer from the vehicle's surroundings, transmitted by the mirror enables us to conduct the virtual tests of the real objects. The necessary data such as: position of eyes points in the real vehicle, the location of the mirrors and their features, and width of the vehicle must be measured in the vehicle and introduced into the program (Fig. 3). The evaluation of fulfilment of the demands by the vehicle is

considerably less strenuous than in the real tests, Fig. 4 and 5.

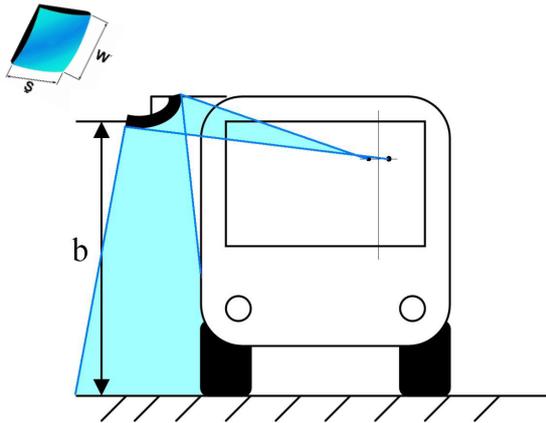
Moreover, the model enables to test sensitiveness to particular variables and determine the limits of the mirrors' use in the indirect visibility of the areas of the vehicle's surroundings. An increase of the extent of the regions which can be observed by the driver with the help of the indirect visibility devices lowers the danger of a collision or accident occurring. It concerns the newly produced vehicles. The vehicles operated so far are the potential source of accidents. The high level of the danger of collision or accident occurrence can be lowered by retrofitting the older vehicles with devices of the visual transfer – Fig. 5.

To estimate the possibilities of taking these actions, the comparative analysis was made.

It relied on determining relationships of minimum dimensions of the mirror (height  $W \times S$  width) on the values of the curvature radius of the mirror sphere taking into consideration the fact that such mirror enables the driver to observe the region which must correspond with the previous regulations and the new ones.

In the case considered, the test object is the system  $K - P - O$  (Driver – Vehicle – Surroundings) and to be precise, the driver observing surroundings of the vehicle through the mirrors, placed on the vehicle, which he is not able to watch directly. The object of the tests is an indirect visual transfer made with the help of the mirrors, being the vehicle's equipment.

The comparative analysis was made by using the above described mathematical model of the tests' object, created by the author and the program in the language C++ developed based on the model with the use of the Open GL library. The relationship between the extent the mirrors cover the regions of classes IV and V, vehicles (which are the trucks' equipment) and the value of the radius of the mirror's curvature, was tested.



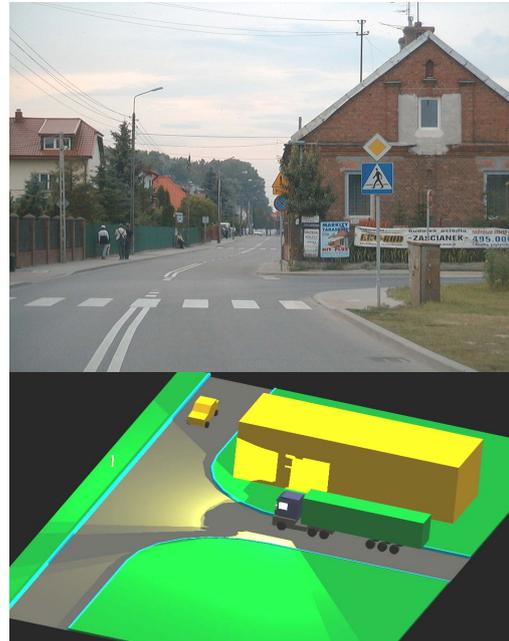
**Fig. 5.** The diagram of the measurement of the height over the road –  $b$ , of the close vision mirror location over the road and the mirror's dimensions and the real measuring stand with the manikin and the halogen lamp head

Everybody should have the possibility to observe a sufficiently long section to be able to perform a braking manoeuvre that stops the vehicle and in this way avoids a collision. When the natural hindrances that obscure the view occur, the drivers should be assisted for example by mirrors. These situations occur in the places where there is a building (colliding with the widening of the road) which creates a potential area for the accident threat – Fig. 6. The similar impediments appear when in the road's surroundings are the steep hills – for example in the mountainous region – drawing no 7.

If we compare the Polish requirements for the mirrors on the road with the requirements concerning the vehicle mirrors included in UN ECE Regulation No. 46 [3], we can draw the following conclusions:

1. for the mirrors on the road there is no convexity defined for example of the curvature radius of the section of the sphere, cylinder or other surface which the mirror should be. The lack of set standards concerning the kind of convexity of the value of the curvature radius (spherical or cylindrical) creates a situation where each

manufacturer shapes the mirror arbitrarily and not in a unified way. As a result, the traffic participants need a long time to adapt and understand the visual transfer transmitted by this mirror.



**Fig. 6.** The example above reflects the limitation of visibility on the crossing by the building which covers the part of the road for people arriving from the left and the diagram shows an example of this real situation.



**Fig. 7.** The example of the mirrors' installation – from the right and from the left of the steep hills causing the visibility impediment

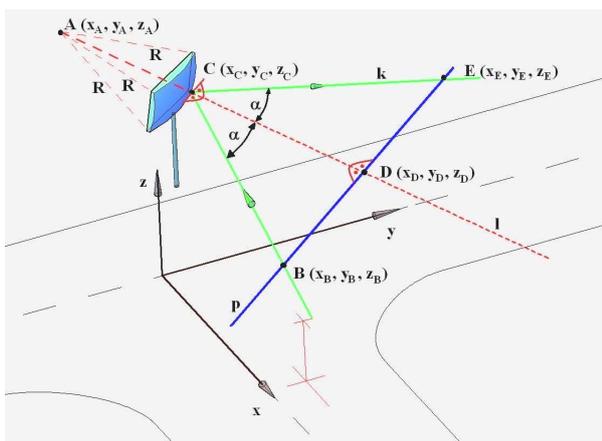
2. as a result of the above there were no values determined of the admissible deviations of the radius of this curvature which determines the disfigurement of the picture transmitted by the mirror – the inability to determine the admissible values of the deviations of the curvature radius on its whole surface admits its great irregularity. Thus

it admits considerable disfigurements of the picture transmitted by this mirror. This extends the time of understanding the information.

3. there was no value determined for the minimum reflection coefficient of the shiny surface, which is particularly important at night – a lack of the minimum value for the coefficient of a shiny surface reflection introduces a complete freedom of choice, which at the small values of this coefficient reduces considerably the visual transfer and impairs the observation as an example in the weak light.

4. a complete freedom of choice introduced as far as the mirror dimensions and shape are concerned causes the existence of the great variety of the picture distortion. This way, the driver needs considerably longer time to receive the visual transfer which is transmitted by the mirror, to understand it and to make a proper decision. The impediments caused by the above can result in making mistakes, thus creating an accident threat. Insufficient training and misunderstanding of information increase the accident threats.

5. new terms were introduced such as: mirrors with increased observation angle, minimum distance of angle observation; the introduced terminology is not defined, unclear and inaccurate. Appropriate application of the regulation - UN ECE to the vehicles accurately determines parameters of the mirror which must be installed on the vehicle to assure possibility of observation of defined regions around the vehicle's surroundings. During testing of the limitation of visual transfer in the particular configuration of the road surroundings - drawing no 6 - the author worked out a mathematical model of this occurrence, in the form of calculation algorithm. Fig. 8 represents a diagram of a model that shows the problem of reflection in the specular reflector for the mirror placed on the road.

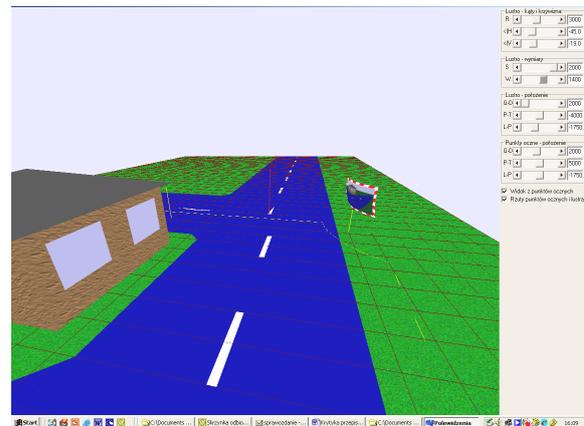


**Fig. 8. Model of the problem of reflection in the specular reflector for the mirror placed on the road**

Setting up, around the vehicle, the area on the surface  $xy$ , for example compatible with the current regulations or expectations, we can determine:

- whether, for a given mirror and its position on the road, the driver can see the area required with his eyesight,
- what conditions must be fulfilled (height, width, curvature radius of the mirror, parameters of mirror placement on the road, adjustment) to enable the driver to see with his eyesight the area required for a given place,
- the limitations for the use of the mirror for the indirect visibility of the road surroundings for a given category of the vehicle and constructional solutions,
- susceptibility of the test object (which is the function of many variables) to the particular variables.

The mathematical model of the visual transfer from the vehicle's surroundings transmitted by the mirrors described above, was helpful to create the calculation program in the C++ programming language with the use of the Open GL library, operating under the Microsoft Windows operating system. On Fig. 9 there is an example of the picture generated by the program which is seen by the program operator on the computer screen.



**Fig. 9. Computer simulation of indirect observation through the mirror on the road**

The algorithm of the picture generating in the program is as follows. The reflected radius was marked (sent from point **B**). The penetration point of the reflected beam with the walls structure (the group of triangles defining the opaque elements in the space) was determined. The result of the above was the point (spatial coordinates) of cutting of the beam (line of reflection) and the colour of the wall with which the cut through took place. For the points lying on the circumference of the mirror, the proper radiuses were drawn. The rest of the points lying on the surface of the mirror were marked with

the colour of the wall with which the cutting of the beam took place.

This way, on the surface of the mirror was created a picture projecting the surroundings seen by the mirror – Fig. 9. In the defined range of the values we can:

- decrease and increase the size of the presented object – it facilitates seeing the details.
- change the direction of the observation.

The operation of the program relies on inputting such values as the vehicle's position, mirror adjustment. Apart from that, being set is the location against the road and vehicle of:

- driver's ocular points,
- regions which should have been seen by the driver,
- mirrors which are considered (their width, height and the curvature radius, locations)

This modelling method allows to run the tests to select the optimum parameters of the mirror and its location on the road to help the traffic participants use the indirect visibility.

## RECAPITULATION AND CONCLUSIONS

The choice for the solutions of the mirrors placed on the roads adopted in the regulations brings about a wide variety of the mirrors used. As a result, for the traffic participants to understand such inconsistent distorted visual transfers becomes a considerable problem.

Unclear and not readily understood picture transferred by the mirror can cause taking wrong actions and in the effect lead to a collision or an accident.

There are no sufficient arguments for admitting such a variety of mirrors' construction when the counter indications exist. There are no precise requirements concerning the mirrors' construction. It is necessary to propose unified solutions and more accurately specificity of the requirements for the mirrors' construction. There also should be uniform principles worked out for the locations of these mirrors in the places with limited direct visibility.

The skill to use the mirrors on the roads should be an element of training within the courses organized for obtaining a driving license. Also, popularising the skills to use the mirrors, should take place, amongst the drivers who got the driving licenses earlier. The care for undisturbed visual transfer as indispensable source of information is the basic element of the active safety system of the traffic participants in the road transport.

## REFERENCES

[1] OLEJNIK K.: Operating problems of buses and trucks – safe reversing. Journal of 17<sup>th</sup> European Maintenance Congress, 11<sup>th</sup> – 13<sup>th</sup> of May 2004 Barcelona – Spain, 343–348.

[2] OLEJNIK K.: Critical analysis of the current traffic regulations concerning visibility from the position of a vehicle driver. Quarterly Motor Transport 2/2003 distributed by Motor Transport Institute, Warsaw, Poland, 69–80.

[3] UN ECE Regulation No. 46. Uniform provisions concerning the approval of devices for indirect vision and of motor vehicles with regard to the installation of these devices.