

OBJECTIVE EVALUATION METHOD OF VEHICLE CRASH PULSE SEVERITY IN FRONTAL NEW CAR ASSESSMENT PROGRAM (NCAP) TESTS

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Paper Number 15-0055

ABSTRACT

In this study, the available metrics for evaluating the crash pulse severity are reviewed and their accessibility is evaluated by using the frontal New Car Assessment Program (NCAP) test data. The linear regression analysis and sled test simulations are conducted. The new approach is proposed to evaluate the full vehicle crash performance by quantifying the crash pulse severity and restraint system performance separately and objectively.

INTRODUCTION

The safety of occupants in a vehicle crash is highly dependent on the performance of vehicle structure and occupant restraint system. In vehicle crash safety, the role of a vehicle structure is absorbing crash energy efficiently as well as protecting the integrity of the occupant compartment. In general, the performance of vehicle structure is described by the occupant compartment intrusion and vehicle crash pulse. Basically, the occupant restraint system is designed based on the performance of the vehicle structure. So, it is desirable to evaluate the performance of vehicle structure objectively and quantitatively.

In frontal vehicle crash tests, occupant compartment intrusion and vehicle crash pulse are the most fundamental responses of a vehicle's structure. The occupant compartment intrusion is considered as an objective metric for quantifying the deformation severity of a vehicle structure. In general, a large compartment intrusion increases the injury probability of lower extremity of occupants. The vehicle crash pulse is the time history of vehicle acceleration and is used to calculate the changes of velocity and dynamic crush of a vehicle by integration. The vehicle crash pulse is closely related to the head and chest injuries of occupants. However, the severity of the vehicle crash pulse is difficult to be quantified objectively because the injury responses of dummy head and chest are also closely associated with restraint system performance. The crash pulse severity should be an objective measure of how severely the vehicle crash pulse has an effect on the occupant injury. Basically, it is regarded that less severe crash pulses possibly lead to less severe occupant injury.

Recently, there have been many vehicle safety research activities by re-designing current vehicles. For example, a current vehicle is light-weighted by using light-weight materials, and enforced structurally to meet the requirements of new regulatory tests (e.g. IIHS small-overlap frontal test and NHTSA oblique frontal test). When a current vehicle is re-designed, its crash pulse is changed accordingly and existing restraint system is not performing as it was designed any more. Then, it is difficult to conclude how its crash performance gets better or worse than the original one, especially in terms of crash pulse.

Many metrics were introduced and utilized to evaluate the severity of vehicle crash pulse [1-11]. Those metrics are derived from vehicle crash pulse in the frontal impact and can be categorized into 4 groups in the way of how occupant responses are considered. However, their assessability of the crash pulse severity is still uncertain. The objective of this study is to evaluate the assessability of available metrics for quantifying vehicle crash pulse severity in front crash. The vehicle crash pulses of the front New Car Assessment Program (NCAP) tests are utilized. Also, a new approach to evaluate the full vehicle crash performance is proposed.

REVIEW OF EXISTING METRICS

In this study, some of existing metrics for evaluating vehicle crash pulses in frontal vehicle crash are reviewed. The existing metrics can be categorized into four groups: (1) metrics based on vehicle crash pulse only, (2) metrics based on vehicle crash pulse with assumed occupant response, (3) metrics based on vehicle crash pulse with actual

occupant response, and (4) combined metrics with the aforementioned metrics. The velocity $v(t)$ and displacement $d(t)$ are obtained by integration and double integration of the acceleration $a(t)$, respectively.

Category1: Metrics Based on Vehicle Crash Pulse Only

In the category1, the metrics are obtained from the vehicle crash pulse only. Dummy responses in the test are not considered. Therefore, these metrics are independent of occupant restraint system and represent an objective, quantified value of the vehicle crash pulse. However, they can hardly predict dummy responses.

- **Maximum acceleration** $(a)_{\max}$ is simply the maximum value of a vehicle acceleration curve over the duration of the crash event.
- **Moving average acceleration** $(\bar{a})^{\Delta t}$ is calculated as

$$(\bar{a})^{\Delta t} = \frac{1}{\Delta t} \int_t^{t+\Delta t} a(\tau) d\tau, \quad (1)$$

where t is time and Δt is a moving time interval. If Δt is the duration of the crash event, the moving average acceleration becomes the average acceleration. The upper bar indicates the average value. In general, **maximum moving average acceleration** $(\bar{a})_{\max}^{\Delta t}$ is used.

- **Delta-V** ΔV is the total vehicle velocity change over the duration of the crash event, as expressed by

$$\Delta V = (v)_{t=0} - (v)_{\min}. \quad (2)$$

- **Time To Zero Velocity** (TTZV) $(t)_{v=0}$ is the time when vehicle velocity becomes zero.
- **Maximum dynamic displacement** $(d)_{\max}$ is simply the maximum value of a vehicle displacement curve over the duration of the crash event.

Category2: Metrics Based on Vehicle Crash Pulse with Assumed Occupant Response

In the category2, an occupant restraint system is assumed and the metrics are derived from the dummy responses with the assumed restraint system under a given vehicle crash pulse. So, these metrics are independent of actual dummy responses in tests, but they are dependent on a virtual, uniform restraint system. Hence, they represent the objective quantified value of vehicle crash pulse and can predict dummy responses.

In general, the vehicle crash model and the restraint system are simplified in the category2. The common simplified model is a Spring-Mass (SM) system as shown in Figure1. In the SM model, the occupant is assumed as a point mass and the restraint system is a simple spring system. Subscripts V and O stand for vehicle and occupant, respectively. M is the vehicle mass, m is the occupant mass, k is the spring stiffness, and δ is the initial slack between the occupant and restraint system. The upper wave indicates the prescribed motion which is a given vehicle crash pulse. In the actual crash test, the spring is highly nonlinear to represent the operation of seatbelt and airbag.

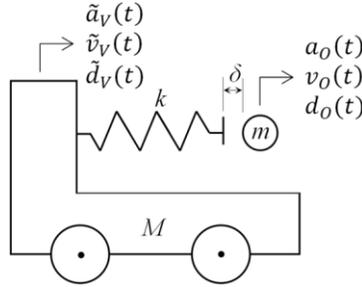


Figure1. The model of a simple Spring-Mass (SM) system.

- The Equation Of Motion (EOM) of the SM model is defined as

$$a_o(t) + \omega_n \cdot d_o(t) = \tilde{a}_v(t), \quad (3)$$

where $\omega_n = \sqrt{k/m}$. The analytical solution of the EOM a_o^{SM} is in the form of the convolution integral expressed as

$$a_o^{SM}(t) = v_v(0) \cdot \omega_n \int_0^t \tilde{a}_v(\tau) \cdot \sin\{\omega_n(t - \tau)\} d\tau. \quad (4)$$

- In the flail-space model [1], the spring stiffness k is assumed to be zero, which indicates that no restraint system is present. So, the occupant moves freely. The allowable moving distance of the point mass is assumed to be 0.6 m. At the instant of occupant impact with the occupant compartment interior, the largest difference in velocity is termed the Occupant Impact Velocity (OIV). Once the impact with the interior occurs, the occupant is assumed to remain in contact with the interior and to be subjected to any subsequent vehicular acceleration.

Figure2 shows the velocity curves in a frontal NCAP test. The black curve is the test vehicle velocity and the red curve is the test occupant (chest) velocity. The occupant is restrained by a certain restraint system. In special cases, it can be assumed that the occupant velocity is prescribed, like the blue dot curve in Figure2, by a special restraint system. In this special case, the occupant translates freely with the initial velocity $v_o(0)$ until the point A. The point A represents the distance of the initial slack δ . This phase is called free flight. After reaching the point A, the occupant is decelerating with a constant acceleration a_o until it reaches the point B. At the point B, the relative velocity $v_{o/V}$ of the occupant to the vehicle becomes zero. This phase between the point A and the point B is called ideal restraint because the occupant has the constant minimum acceleration under a given crash pulse. So, this prescribed occupant velocity is the ideal velocity of the occupant in frontal crash and this special restraint system can be considered as the ideal restraint system. Compared to the SM model, the spring stiffness k will be nonlinear to maintain the constant deceleration of the occupant.

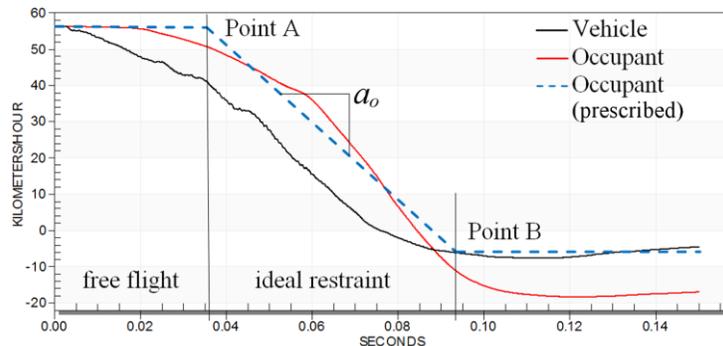


Figure2. Prescribed occupant response in the NCAP test.

- In the **Occupant Load Criterion** (OLC) metric [2], it is assumed that the initial slack δ is 65 mm and the distance between the vehicle and the occupant at the point B is an additional 235 mm. Then, given the point A and the point B, the constant acceleration a_o becomes the critical occupant response which is called the OLC (G). Basically, the OLC means the minimum occupant acceleration induced by a given crash pulse under the protection of the ideal restraint system.
- In the **Maximum Chest Travel** (MCT) metric [3], it is assumed that the initial slack δ and the constant acceleration a_o are predefined. Then, the distance between the point A and the point B is the critical occupant response. This distance is called the MCT (mm).

Category3: Metrics Based on Vehicle Crash Pulse with Actual Occupant Response

In the category3, metrics are obtained from both vehicle crash pulse and actual dummy responses in the test. The metrics are dependent on the dummy responses and restraint system performance in tests. Basically, those metrics identify the contribution of restraint system performance to the full vehicle crash performance. So, they quantify the vehicle structure performance in terms of crash pulse and the restraint system performance, but they are not the objective, quantified value. In general, they are in percentage terms.

- Occupant restraint performance during vehicle deceleration is measured as the relative velocity of the occupant in vehicle divided by the maximum velocity change of the vehicle which is ΔV . This ratio is called the **Restraint Quotient** (RQ) [4] expressed as

$$RQ_C = \frac{v_{OC/V}}{\Delta V}, \quad (5)$$

where

$$v_{OC/V} = v_{OC} - v_V \quad (6)$$

and the subscript C stands for chest. It normally varies between 0 and 1. A RQ value of 0 represents an occupant rigidly coupled to the vehicle interior and a value of 1 indicates that the occupant attains the total velocity change of the vehicle before impacting the vehicle interior. The lower the RQ, the better the restraint performance in a crash. The relative kinetic energy per unit mass is calculated using the maximum relative occupant velocity normalized by a velocity of 5 m/s, which is called **kinetic energy factor** (E) [4] expressed as

$$E_C = \frac{(v_{OC/V})_{\max}^2}{5^2}. \quad (7)$$

- In the SM model in Figure1, the energy per unit mass (or energy density) of an occupant can be expressed as

$$e = \int_0^{d_v} a_o(x) dx_o = \int_0^{d_v} a_o(x) d(x_V + x_{O/V}) = e_{rd} + e_{rs}, \quad (8)$$

where

$$e_{rd} = \int_0^{d_v} a_o(x) dx_V, \quad (9)$$

$$e_{rs} = \int_0^{d_v} a_O(x) dx_{O/V} , \text{ and} \quad (10)$$

$$x_{O/V} = x_O - x_V . \quad (11)$$

e_{rd} is called the ride-down energy density and e_{rs} is the restraint energy density [5-10]. The ride-down energy is attributed to the crush of the front structures of the car and the restraint energy is dissipated by the crushing of the restraint system components. Then, the ride-down efficiency μ is obtained from

$$\mu = \frac{(e_{rd})_{\max}}{\{v(0)\}^2/2} . \quad (12)$$

This metrics reflect the percentage of total kinetic energy absorbed by the vehicle structure.

Category4: Combined Metrics Using the Aforementioned Metrics

In the category4, the metrics are defined as the linear combination of the aforementioned metrics. Mostly, certain metrics are combined to improve better prediction of occupant injury.

- In the Expanded ΔV [11], ΔV is expanded by combining with other metrics. Three expanded ΔV metrics were proposed as

$$\text{Expanded } \Delta V-1 = a_1 \Delta V + a_2 \mu + a_3 E_c , \quad (13)$$

$$\text{Expanded } \Delta V-2 = a_1 \Delta V + a_2 (\bar{a})_{\max}^{\Delta t} + a_3 RQ_c , \quad \text{and} \quad (14)$$

$$\text{Expanded } \Delta V-3 = a_1 \Delta V + a_2 (\bar{a})_{\max}^{\Delta t} + a_3 E_c , \quad (15)$$

where a_1, a_2 , and a_3 are coefficients.

- In the OLC++ [2], OLC was augmented as

$$\text{OLC++} = a_1 \text{OLC} + \frac{a_2}{(t)_{v=0}} + a_3 (\bar{a})_{\max}^{\Delta t} , \quad (16)$$

where a_1, a_2 , and a_3 are coefficients.

DATA ANALYSIS

The aforementioned metrics' assessability of crash pulse severity is evaluated using frontal NCAP test data. A total of 60 frontal NCAP test data, collected from the MY 2012 vehicle test program, are analyzed. The linear regressions of each pair of all metrics and their R^2 values are examined. The larger value of R^2 indicates better fits. It is considered that the pair of two metrics has a linear correlation if the R^2 is greater than 0.5.

It seems that the metrics in Category2 are the fairly appropriate metrics for evaluating the crash pulse severity since they are the objective metrics associated with both vehicle crash pulse and uniform restraint system. So, the relationship between the metrics in Category2 and other metrics are investigated.

Table1 summarizes the linear regression results between Category1 and Category2. The R^2 values of each pair are shown in Table1. It is observed that the OLC and MCT have relatively high R^2 values with maximum acceleration,

maximum moving average acceleration, TTZV, and maximum dynamic crush. However, OIV has low R^2 values. Especially, Delta-V has very low R^2 values with OLC and MCT. Interestingly, although the metrics in Category1 are purely obtained from vehicle crash pulse only without dummy response information, the metrics in Category1 have a good linear correlation with the OLC and MCT in Category2.

Table1. Linear regression results between Category1 and Category2 (highlighted cell indicates that R^2 is greater than 0.5).

R^2	OIV	OLC	MCT
Max. Accel.	0.214	0.527	0.445
Max. Moving Average Accel. ($\Delta t=25\text{msec}$)	0.477	0.735	0.607
Delta-V	0.395	0.002	0.029
TTZV	0.281	0.859	0.793
Max. Dynamic Crush	0.051	0.678	0.704

Table2 summarizes the linear regression results between Category2 and Category3. It is observed that the OLC and MCT have high R^2 values with the metrics in Category3, but OIV has very low R^2 values. Basically, the metrics in Category3 describe the actual effects of the vehicle crash pulse and restraint system on the dummy responses in the test. So, the high linear correlation between OLC and MCT in Category2 and the metrics in Category3 indicates that the OLC and MCT are able to predict the effect of the vehicle crash pulse on the dummy response and assess the crash pulse severity adequately.

Table2. Linear regression results between Category2 and Category3 (highlighted cell indicates that R^2 is greater than 0.5).

R^2		OIV	OLC	MCT
Driver	RQ _c	0.051	0.669	0.679
	E _c	0.098	0.705	0.680
	Ride-Down Efficiency	0.058	0.617	0.624
Passenger	RQ _c	0.056	0.515	0.504
	Ride-Down Efficiency	0.120	0.474	0.447

Table3 summarizes the linear regression results between the aforementioned metrics and dummy injuries in the NCAP tests. It is observed that the R^2 values of all pairs between metrics and dummy injuries are very low. The Delta-V is commonly used to address the crash severity, but it can hardly predict the dummy injuries as well.

Table3. Linear regression results between metrics and occupant injury responses.

R^2	Driver (H3 50% male)			Passenger (H3 5% female)			
	HIC ₁₅	Chest Peak Accel.	Chest Peak Deflection	HIC ₁₅	Chest Peak Accel.	Chest Peak Deflection	
Max. Accel.	0.000	0.043	0.057	0.003	0.125	0.015	
Max. Moving Average Accel. ($\Delta t=25\text{msec}$)	0.002	0.139	0.064	0.013	0.284	0.005	
ΔV	0.003	0.007	0.019	0.005	0.001	0.055	
TTZV	0.005	0.101	0.060	0.018	0.339	0.016	
Max. Dynamic Crush	0.022	0.055	0.036	0.017	0.224	0.025	
OIV	0.023	0.070	0.039	0.001	0.186	0.002	
OLC	0.014	0.090	0.023	0.026	0.342	0.005	
MCT	0.021	0.075	0.016	0.026	0.305	0.012	
Driver	RQ _c	0.009	0.086	0.056	0.028	0.178	0.069
	E _c	0.013	0.084	0.044	0.030	0.196	0.042
	Ride-Down Efficiency	0.010	0.113	0.052	0.052	0.201	0.081
Passenger	RQ _c	0.002	0.036	0.034	0.026	0.109	0.075
	Ride-Down Efficiency	0.001	0.106	0.067	0.020	0.137	0.120
Expanded ΔV -1	0.012	0.082	0.037	0.024	0.192	0.030	
Expanded ΔV -2	0.005	0.110	0.053	0.022	0.220	0.042	
Expanded ΔV -3	0.010	0.096	0.043	0.025	0.217	0.030	
OLC++	0.014	0.092	0.043	0.019	0.310	0.010	

Based on the linear regression of the metrics shown in Table1 and Table2, it is found that some of metrics fairly can assess the crash pulse severity. Especially, it seems that the OLC has the high accessibility of crash pulse severity according to its high linear correlation to many metrics in Category1 and Category3. However, none of metrics can predict dummy injuries. Every vehicle has its own uniquely designed restraint system, and the dummy responds very

sensitively to various restraint system performances. Moreover, crash tests have very high dispersion errors in dummy injuries in general. In these circumstances, it is very difficult to predict dummy injuries in frontal crash using existing metrics.

SLED TEST SIMULATIONS

In order to identify the effect of vehicle crash pulse and restraint system on dummy responses, sled test simulations with Hybrid III 50th male dummy FE model are conducted using all different 60 NCAP crash pulses. Two cases are considered for each sled test simulation; (1) fix dummy clearance dimensions and (2) adjust some of dummy clearance dimensions, such as CS (chest to steering hub), SCA (steering column angle), and KD (knee to dash). The uniform generic restraint system (seatbelt and airbag) is utilized for all sled test simulations. The dummy responses in sled test simulations are monitored.

Figure3 shows the linear regressions of chest peak accelerations in NCAP tests and sled test simulations. They show little correlation between two tests. Since the sled test simulations utilize the NCAP crash pulses, main difference between two tests is that all the different restraint systems are used in NCAP tests and one uniform restraint system is used in sled test simulations. It can be interpreted as the data dispersion is mainly caused by the various restraint system performances in the NCAP test vehicles.

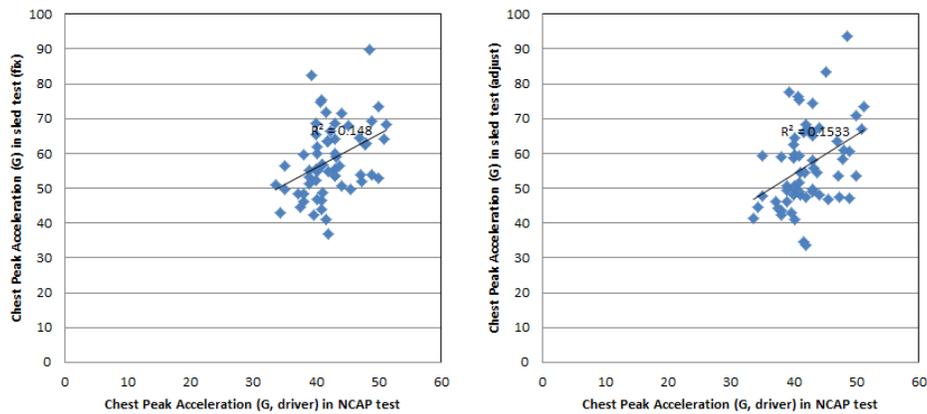


Figure3. Data distribution of chest peak acceleration in NCAP tests vs. chest peak acceleration in sled test simulations

Figure4 shows the linear regressions of chest peak accelerations in sled test simulations and the OLC metric. They show high correlation between two metrics. In other words, the OLC metric is able to predict dummy responses and injuries if the uniform restraint system is used in all test vehicles. Also, it can be observed that the different dummy clearance dimension makes the degree of data dispersion increase, but the linear correlation is still high.

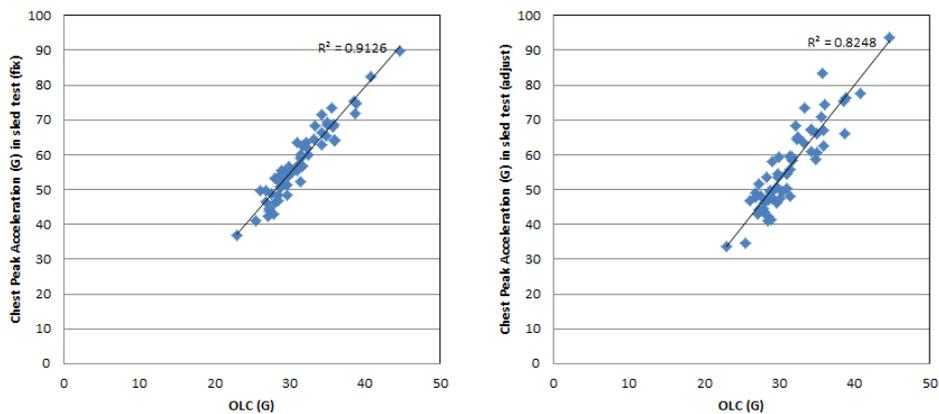


Figure4. Data distribution of OLC vs. chest peak acceleration in sled test simulations

DISCUSSION

It seems that the Category3 provide an ideal approach to evaluate a full vehicle crash performance by quantifying both the vehicle structure performance and restraint system performance separately, where the vehicle structure performance includes only the crash pulse severity, but not the occupant compartment intrusion. However, the metrics in the Category3 are objective. The performance of the restraint system in vehicles are various in frontal crash tests. Moreover, the restraint system performance is dependent on the crash pulse severity, which means that the restraint system performance is coupled with crash pulse severity. So, it is difficult to quantify the restraint system performance objectively by de-coupling from the effect of a vehicle crash pulse. In this study, a new approach is proposed to evaluate the full vehicle crash performance by quantifying the crash pulse severity and restraint system performance separately and objectively. The new approach makes the restraint system performance de-coupled from the effect of the vehicle crash pulse.

Figure5 shows the three datasets of occupant chest peak accelerations with respect to maximum moving average acceleration of vehicles. The first dataset, red squares, is the OLC values. This dataset shows good linear correlation with high R^2 value. Basically, the OLC metric indicates the minimum occupant acceleration under a given crash pulse. Therefore, the OLC data forms the lower boundary in Figure5. The second dataset, green triangles, is the occupant peak accelerations obtained from the analytical solution (Eq. 4) of the SM model with the constant spring stiffness k . This dataset also shows good linear correlation with high R^2 value. Because the restraint system is regarded as a simple linear spring, the occupant response in the SM model under a given crash pulse is likely worse than the one with actual restraint system. Maybe the occupant response in the SM model with the linear spring will be the worst under the given crash pulse. Hence, it can be seen that the occupant peak acceleration in the SM model forms the upper boundary in Figure5. The linear regression slopes of two datasets (red squares and green triangles) are actually very close. So, the OLC metric with respect to maximum moving average acceleration of vehicles is considered as the crash pulse severity metric and the crash pulse severity index is defined by normalizing the crash pulse severity metric.

In Figure5, the third dataset, blue rhombuses, is the chest peak accelerations (driver) in the frontal NCAP tests. These data points are distributed between the lower and upper boundaries formed by two datasets (red squares and green triangles). Some data points are close to the lower or upper boundaries, that is, those data points are close to their minimum or maximum values in their crash pulse severity levels. The rational explanation of the data dispersion between two boundaries is because the different restraint system performance in every vehicle in the frontal NCAP tests. So, in order to de-couple the restraint system performance from the crash pulse effect, the third dataset is mapped to the OLC-axis plane and normalized to generate the restraint system performance index.

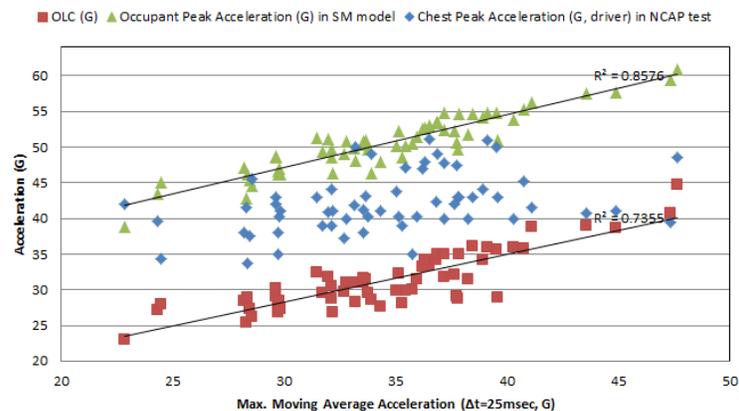


Figure5. Data distribution of max. moving average acceleration vs. OLC, occupant peak acceleration in SM model, and chest peak acceleration (driver) in NCAP test.

The crash pulse severity index and restraint system performance index are objective and independent each other. Two indices describe the full vehicle crash performance in the frontal NCAP test. Figure6 shows the data distribution of the full vehicle crash performance in two indices plane, which is very informative. For instance, the point A in Figure6 means low crash pulse severity but poor restraint system performance, and the point B indicates

high crash pulse severity but good restraint system performance. Practically, when the vehicle crash pulse is known, the plot in Figure5 shows the crash pulse severity and the range of the dummy chest peak acceleration, and the plot in Figure6 tells the performance of the current vehicle's restraint system in the frontal crash.

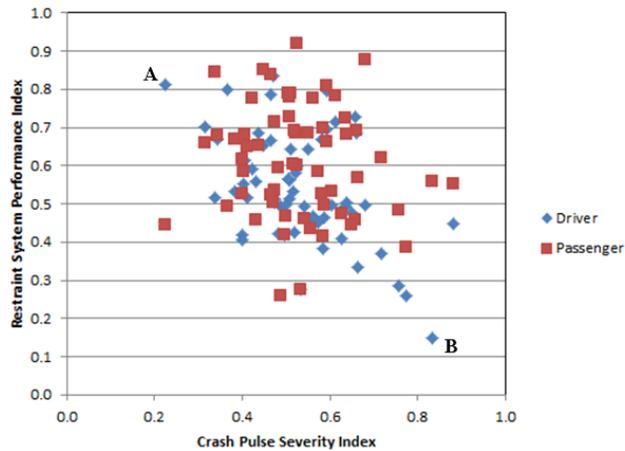


Figure6. Data distribution of crash pulse severity index vs. restraint system performance index.

CONCLUSIONS

In this study, the existing metrics for evaluating the crash pulse severity are reviewed and categorized into four groups: (1) metrics based on vehicle crash pulse only, (2) metrics based on vehicle crash pulse with assumed occupant response, (3) metrics based on vehicle crash pulse with actual occupant response, and (4) combined metrics with the aforementioned metrics.

Their accessibility of crash pulse severity is evaluated by using the frontal NCAP test data. A total of 60 frontal NCAP test data, collected from the MY 2012 vehicle test program, are analyzed. The linear regression analysis shows that some of metrics fairly can assess the crash pulse severity. Especially, it seems that the OLC has the high accessibility of crash pulse severity according to its high linear correlation to many metrics in Category1 and Category3. However, none of metrics can predict dummy injuries.

The sled test simulations are conducted using the NCAP pulses. The uniform generic restraint system (seatbelt and airbag) is utilized for all sled test simulations. The results conclude that the various restraint system performances in the NCAP test's vehicles cause a big variation in dummy responses and make it difficult to predict dummy injuries in the frontal NCAP test.

The new approach is proposed to evaluate the full vehicle crash performance in the frontal NCAP test by quantifying the crash pulse severity and restraint system performance separately and objectively. The crash pulse severity index is defined by normalizing the OLC metric, and then the restraint system performance index is defined by decoupling the restraint system performance from the crash pulse effect. Two indices describe the full vehicle crash performance in the frontal NCAP test. The new approach provides a quantitative and objective way to analyze the crash performance of a vehicle in the frontal NCAP test.

ACKNOWLEDGEMENTS

The authors are grateful to Eun-Young Kim for data processing.

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