AUTOMATIC CONTROL OF VEHICLE STEERING SYSTEM DURING LANE CHANGE

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ABSTRACT

Mechatronic systems assist drivers in safe driving of cars more and more often. A vision of a totally automated car realizing many manoeuvres without driver's participation becomes closer and closer. The lane change manoeuvre is one of the basic manoeuvres on the ground of which sequences of complex manoeuvres can be composed, e.g. vehicle passing or obstacle avoidance manoeuvre. For those reasons, automation of lane change manoeuvre appears to be essential for automation of vehicle driving and is a subject of numerous research studies. Within a research project, the authors have undertaken extensive analytical studies on application of active steering system EPS in automatic driving of a two-axis truck equipped with typical elements of ESC system and obstacle detectors, as well as road monitoring systems.
The present paper focuses on theoretical aspects of the synthesis of an automatic controller for the EPS active steering system. Simulation studies of an automatically controlled lane change manoeuvre illustrate the application of the methodology. The basis for theoretical considerations and numerical studies is the mathematic model of the controlled system (vehicle) and the controller. A complex, detailed description of the dynamics of a two-axis truck, taking into account nonlinearities and vehicle motion in 3D space, is included in the simulation model. The model of the controller is based on a reference model which is significantly simplified and hence is highly effective for carrying out necessary computations in real time. An algorithm of the controller operating as a Kalman regulator in a closed loop system is developed on the basis of this model. The time decomposition of the automatic control process into two phases –lateral displacement of the vehicle and stabilization of its position – is an essential, original distinguishing feature of the algorithm. Thanks to this decomposition, the structure of the control system is relatively simple. Feedback signals provided by the sensors available in a typical ESC system (lateral acceleration, yaw velocity) are used in the control process. The vehicle reference model and resulting control algorithms are presented in the paper. Simulation results refer to a two-axis truck travelling with a constant velocity on a straight, uniform road. At certain time instant the vehicle starts executing the lane change manoeuvre. Simulations were carried out for a number of cases with varying model parameters. That allowed estimating the sensitivity of the control algorithm to both perturbations of vehicle's physical and operational parameters and to perturbations of parameters related to the obstacle. The results of simulations show that the proposed concept of the vehicle automatic control performs well in computational tests. The method of automatic execution of the lane change manoeuvre presented in the paper can offer an attractive alternative for vehicle control engineers and researchers working in the fields of active steering systems of vehicles, including commercial trucks.

INTRODUCTION

Development of modern automotive technology results in an increasing use of mechatronic systems assisting drivers in safe driving of road vehicles. The prospect of a totally autonomous vehicle performing a range of manoeuvres without driver's participation nears. The lane change manoeuvre is one of basic manoeuvres on the basis of which more complex manoeuvres such as vehicle overtaking or obstacle avoidance can be undertaken. Obstacle avoidance is necessary when an object suddenly appears on the vehicle path in a distance smaller than the estimated travel needed to stop. For these reasons, automation of the lane change manoeuvre is essential to achieve the goal of fully autonomous vehicle control. It is a subject of numerous research programs as well as prototype development and testing.
Publications on automation of lane change manoeuvre usually refer to the concept of automatic control including optimal path planning and then trajectory tracking [2, 4, 9, 10, 11, 12]. Trajectory planning is sometimes treated as a problem of parametric optimization of heuristically assumed forms of the desired path (segments of sinusoidal function, composition of arcs, line segments, parabola segments etc.). Optimization of the desired path should not only achieve short manoeuvre duration, desired smoothness of the trajectory, limitation of side jerks, but also ensure that the planned path will be feasible for efficient trajectory control. As known, trajectory tracking errors depend on

the quality of the desired path. Trajectory tracking controllers proposed in publications cited above are based on known algorithms from the control theory. Obviously, parameters of the vehicle, as well as parameters of its steering system, have significant influence on the optimal path and the choice of the tracking controller [13, 14].
Within the research project [7], comprehensive analytical studies have been carried out on the application of the active steering system EPS (Electric Power System) in automatic driving of a commercial truck of medium load capacity. Considered traffic situations included suddenly appearing obstacle potentially causing collision. The truck under consideration was equipped with typical elements of ESC (Electronic Stability Control) and obstacle detectors, as well as road monitoring systems.
This paper is based on a portion of the aforementioned studies [7]. It focuses on theoretical aspects of the synthesis of an automatic controller for the EPS active steering system. Simulation studies of an automatically controlled lane change manoeuvre illustrate the application of the methodology.
The basis for theoretical considerations and numerical studies is the mathematical model of the controlled system and the controller. A complex, detailed description of the dynamics of a two-axis truck, taking into account nonlinearities and vehicle motion in 3D space, is included in the simulation model representing the real system to be controlled. Complex nonlinear dynamics of the steering mechanism is included in the model, with free play and friction in the joints. The model of the controller is based on a simple reference model – known in the literature as a bicycle model of a car. Desired path as well as the structure and parameters of the controller are determined using this model.
An essential, distinguishing feature of the control algorithm is the decomposition of the lane changing manoeuvre into two phases: vehicle turning to move swiftly into the adjacent lane and then stabilization in the direction of the new lane. Owing to this decomposition, the structure of the control algorithm is relatively simple. Feedback signals provided by the sensors available in a typical ESC system (lateral acceleration, yaw velocity) are used in the control process. The vehicle reference model and resulting control algorithms are presented in the paper. Simulation results refer to a two-axis commercial vehicle of medium load capacity travelling with a constant velocity on a straight, uniform road. At certain time instant the vehicle starts executing the lane change manoeuvre. Simulations were carried out for a number of cases with varying model parameters. That allowed estimating the sensitivity of the control algorithm to both perturbations of vehicle's physical and operational parameters and to perturbations of parameters related to the obstacle. The results of simulations show that the proposed concept of the vehicle automatic control performs well in computational tests.

LANE CHANGE MANOEUVRE IN THE FRAMEWORK OF THE CONTROL THEORY

Assumptions

An obstacle suddenly appears on a straight stretch of the road in front of the travelling vehicle. The vehicle starts braking (manual or automatic process) until the time instant the control system discovers that further braking inevitably leads to a crash. After automatic monitoring of obstacle surroundings, if conditions allow for that, the rotation of the steering wheel is activated automatically in order to avoid the obstacle while travelling with a constant velocity that was reached at the final stage of braking. At the end of the manoeuvre vehicle should travel on the lane parallel to the primary lane. In that way, the control of the vehicle moves from the braking phase to the lane changing phase.
The purpose of the study presented in this paper is to show the concept of the steering wheel controller that would execute the lane change manoeuvre once it was automatically initiated. The method of setting controller parameters knowing the parameters of the vehicle reference model and allowable variable ranges will also be presented. The often challenging strategy of decision making to activate the automatic lane change operation will not be considered.

Strategy for Steering Wheel Rotation Control

The control task involves two output variables – displacement of the centre of mass and angular position of the vehicle body in relation to the trajectory of the centre of mass – and is to be carried out with one control input (steering wheel rotation). It is convenient to divide the process of steering wheel control into two successive phases of "lateral displacement" and then "stabilization".
Realization of the task of vehicle lateral displacement allows for some level of vehicle yaw with respect to the roadway axis after required lateral displacement of the vehicle centre of mass has been achieved, ensuring obstacle avoidance with certain safety margin. The priority is on the fast execution of this phase of the manoeuvre.

The task of stabilization involves adjusting the angular orientation of the vehicle to become parallel to the roadway axis. Here, the priority is on accurately performing the procedure to ensure that the vehicle follows the new lane.
Such decomposition of the control tasks is consistent with practice of driving a car by experienced race drivers. Control in the first phase of the process can be carried out partly in an open loop (“blindly”, “as quickly as possible”) by generating an appropriate steering wheel rotation. Accuracy at this phase of the manoeuvre is ensured by the use of the earlier identified reference model. An additional corrective control will also be present during this phase. Namely, the correction of the steering wheel rotation angle is to be carried out in a closed loop, through the comparison of the lateral displacement of the vehicle (according to the reference model) with the measured position. Control in the second phase must be carried out completely in the closed loop, by regulating the angular orientation of the vehicle with respect to the roadway axis.
The concept of the control strategy based on the decomposition into two phases is shown in Fig.1.

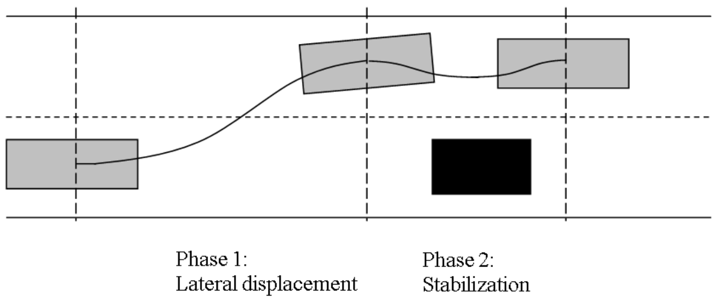


Figure 1. Decomposition of the lane changing manoeuvre.

Theoretical validation of this control strategy is presented in the subsequent part of the paper.

The Reference Model

Theoretical considerations will be conducted using the bicycle model of a car. The model describes the lateral dynamics of the vehicle travelling with constant velocity in the presence of small disturbances. This model is used in many studies related to the automatic control of vehicles. The bicycle model concept is illustrated in Fig. 2.

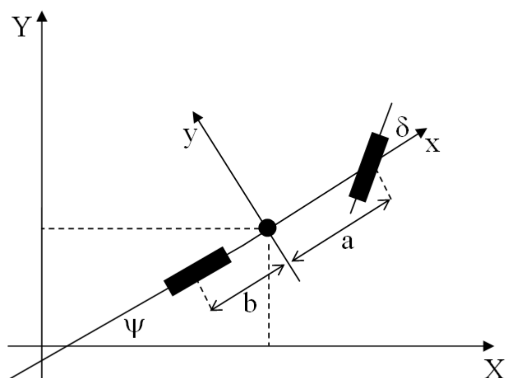


Figure 2. Concept of the bicycle model of a car.

Nomenclature of model variables and parameters:
t – time (t = 0 denotes the start of control),
$\delta(t)$ – steer angle of front wheels,
$\psi(t)$ – vehicle yaw angle,
$\Omega(t)$ – yaw velocity of the vehicle ($\Omega(t) = \dot{\psi}(t)$),
U(t) – lateral velocity of the vehicle in the local coordinate frame,
V – longitudinal velocity of the vehicle (constant) in the local coordinate frame,
X(t), Y(t) – vehicle centre of mass coordinates in the global coordinate frame,

m – mass of the vehicle,
J – moment of inertia of the vehicle with respect to the vertical axis passing through its centre of mass,
a, b – distances from the front and rear wheel axes, respectively, to the projection of the centre of mass,
k_A, k_B – cornering stiffnesses at the front and rear wheel centres, respectively.

The mathematical model for the subsequent formulation is represented by linearized equations of motion derived from the balance of forces and moments acting on a two-wheeled vehicle. The equations expressed in the moving coordinate frame are:

$$m\dot{U}(t)+\frac{k_A+k_B}{V}U(t)+\frac{mV^2+k_Aa-k_Bb}{V}\Omega(t)=k_A\delta(t) \tag{1}$$

$$J\dot{\Omega}(t)+\frac{k_Aa^2+k_Bb^2}{V}\Omega(t)+\frac{k_Aa-k_Bb}{V}U(t)=k_Aa\delta(t) \tag{2}$$

Transformation from the moving coordinate frame to the frame connected to the road is described by the following relations:

$$\psi(t)=\int_0^t\Omega(\tau)d\tau \tag{3}$$

$$\dot{X}(t)=V\cos(\psi(t))-U(t)\sin(\psi(t)) \tag{4}$$

$$\dot{Y}(t)=V\sin(\psi(t))+U(t)\cos(\psi(t)) \tag{5}$$

Trajectory of the vehicle's centre of mass Y(X) can be determined from the following relationships:

$$X(t)=\int_0^t\dot{X}(\tau)d\tau=\int_0^t\left(V\cos(\psi(\tau))-U(\tau)\sin(\psi(\tau))\right)d\tau \tag{6}$$

$$Y(t)=\int_0^t\dot{Y}(\tau)d\tau=\int_0^t\left(V\sin(\psi(t))+U(\tau)\cos(\psi(\tau))\right)d\tau \tag{7}$$

The relationships (1-7) will be treated as an initial reference model of the vehicle.
With small and short time duration disturbances occurring during obstacle avoidance it is allowed to use linearized form of the transformation equations. Applying the Taylor series approximation gives:

$$\cos(\psi(t))\approx 1 \qquad \sin(\psi(t))\approx\psi(t) \tag{8, 9}$$

$$U(t)\sin(\psi(t))\approx 0 \qquad U(t)\cos(\psi(t))\approx U(t) \tag{10, 11}$$

Therefore, on the basis of (4-5):

$$\dot{X}(t)=V \tag{12}$$

$$\dot{Y}(t)=V\psi(t)+U(t) \qquad \ddot{Y}(t)=V\dot{\psi}(t)+\dot{U}(t) \tag{13, 14}$$

and also:

$$U(t)=\dot{Y}(t)-V\psi(t) \qquad \dot{U}(t)=\ddot{Y}(t)-V\dot{\psi}(t) \tag{15, 16}$$

After substitution into the equations of motion (1-2) and rearranging one gets:

$$m\ddot{Y}(t)+\frac{k_A+k_B}{V}\dot{Y}(t)+\frac{k_Aa-k_Bb}{V}\dot{\psi}(t)-\left(k_A+k_B\right)\psi(t)=k_A\delta(t) \tag{17}$$

$$J\ddot{\psi}(t)+\frac{k_Aa^2+k_Bb^2}{V}\dot{\psi}(t)-\left(k_Aa-k_Bb\right)\psi(t)+\frac{k_Aa-k_Bb}{V}\dot{Y}(t)=k_Aa\delta(t) \tag{18}$$

The trajectory of the vehicle's centre of mass Y(X) is determined by the relationships:

$$X(t)=\int_0^t\dot{X}(\tau)d\tau=\int_0^tVd\tau=Vt \tag{19}$$

$$Y(t)=\int_0^t\dot{Y}(\tau)d\tau \tag{20}$$

The above developed equations (17-20) will be treated as a simplified reference model of the vehicle.
In the initial reference model, the relationships (1, 2, 3) are linear and therefore can be subjected to the Laplace transformation. Then, at zero initial conditions of variables U(t) and Ω(t), an equivalent notation of the reference model can be defined in the transfer function form with the operator variable s.

$$\tilde{U}(s)=G_{U\delta}(s)\tilde{\delta}(s) \tag{21}$$

$$\tilde{\Omega}(s)=G_{\Omega\delta}(s)\tilde{\delta}(s) \tag{22}$$

$$\tilde{\psi}(s)=\frac{1}{s}\tilde{\Omega}(s) \tag{23}$$

where transfer functions have standard forms:

$$G_{U\delta}(s)=\frac{G_{U\delta 0}(T_{U\delta}s+1)}{T_0^2 s^2+2\xi_0 T_0 s+1} \tag{24}$$

$$G_{\Omega\delta}(s)=\frac{G_{\Omega\delta 0}(T_{\Omega\delta}s+1)}{T_0^2 s^2+2\xi_0 T_0 s+1} \tag{25}$$

The transfer function parameters can be described by the formulae:

$$T_0=V\sqrt{\frac{mJ}{k_A k_B(a+b)^2-MV^2(k_A a-k_B b)}} \tag{26}$$

$$\xi_0=\frac{\left(m\left(k_A a^2+k_B b^2\right)+J\left(k_A+k_B\right)\right)}{2\sqrt{mJ\left(k_A k_B(a+b)^2-mV^2(k_A a-k_B b)\right)}} \tag{27}$$

$$G_{U\delta 0}=\frac{\left(k_A k_B(a+b)b-mV^2 k_A a\right)V}{k_A k_B(a+b)^2-mV^2(k_A a-k_B b)} \tag{28}$$

$$T_{U\delta}=\frac{Jk_A V}{k_A k_B(a+b)b-mV^2 k_A a} \tag{29}$$

$$G_{\Omega\delta 0}=\frac{k_A k_B(a+b)V}{k_A k_B(a+b)^2-mV^2(k_A a-k_B b)} \tag{30}$$

$$T_{\Omega\delta}=\frac{maV}{k_B(a+b)} \tag{31}$$

Operational calculus cannot be applied to Eqs. (4) and (5) due to the presence of nonlinear terms.
All relationships in the simplified reference model are linear and therefore the Laplace transformation can be applied and appropriate transfer functions can be determined. Then, at zero initial conditions of variables $\dot{Y}(t),\psi(t),\ \Omega(t)$, an equivalent notation of the simplified reference model in the transfer function form can be obtained as follows:

$$Y(s)=G_{Y\delta}(s)\delta(s) \tag{32}$$

$$\psi(s)=G_{\psi\delta}(s)\delta(s)=\frac{G_{\Omega\delta}(s)}{s}\delta(s) \tag{33}$$

where

$$G_{Y\delta}(s)=\frac{G_{\Omega\delta 0}V\left(T_1^2 s^2+2\xi_1 T_1 s+1\right)}{s^2\left(T_0^2 s^2+2\xi_0 T_0 s+1\right)} \tag{34}$$

$$T_1=\sqrt{\frac{J}{k_B(a+b)}}\qquad \xi_1=\frac{b}{2V}\sqrt{\frac{k_B(a+b)}{J}} \tag{35, 36}$$

Responses to the Step Input of Wheels Rotation

If $\delta(t)=\delta_0 1(t)$ (1(t) – Heaviside function) (37)

then $\delta(s)=\delta_0\frac{1}{s}$ (38)

$$\lim_{t\to\infty}U(t)=\lim_{s\to 0}(sU(s))=\lim_{s\to 0}(sG_{U\delta}(s)\delta(s))=\lim_{s\to 0}\left(s\frac{G_{U\delta 0}(T_{U\delta}s+1)}{\left(T_0^2 s^2+2\xi_0 T_0 s+1\right)}\delta_0\frac{1}{s}\right)=G_{U\delta 0}\delta_0$$

(39)

$$\lim_{t\to\infty}\Omega(t)=\lim_{s\to 0}(s\Omega(s))=\lim_{s\to 0}(sG_{\Omega\delta}(s)\delta(s))=\lim_{s\to 0}\left(s\frac{G_{\Omega\delta 0}(T_{\Omega\delta}s+1)}{\left(T_0^2 s^2+2\xi_0 T_0 s+1\right)}\delta_0\frac{1}{s}\right)=G_{\Omega\delta 0}\delta_0$$

When the time responses to the step input are available, these formulae can be used for identification of unknown parameters of the reference model.

Steady State Responses Y(t) and ψ(t) to the Sharp Pull of the Wheels in One and Next in the Opposing Direction

The two-sided sharp pull of the steering wheel with a hold time T can be described by a combination of step functions 1(t) of wheels rotation (see Fig. 3):

$$\delta(t)=\delta_0\left(1(t)-2*1(t-T)+1(t-2T)\right) \tag{40}$$

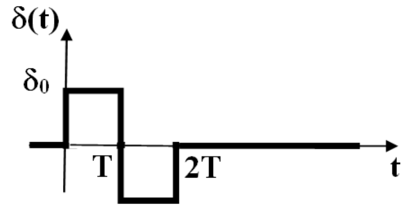


Figure 3. Two-sided sharp pull of the wheels.

An accurate analysis of the time histories of Y(t) and ψ(t) can be carried out on the basis of simulation results. The limits of ψ(t) and Y(t) at t → ∞ can be determined knowing the transform of the input function δ(t) and the appropriate transfer functions. For Y(t), due to the approximate character of the simplified reference model, this process will produce an estimated value.
The Laplace transform of δ(t) defined in formula (40) is given by:

$$\delta(s) = \delta_0\left(\frac{1}{s} - 2\frac{1}{s}e^{-sT} + \frac{1}{s}e^{-s2T}\right) = \delta_0\frac{1-2e^{-sT}+e^{-s2T}}{s} = \delta_0\frac{\left(1-e^{-sT}\right)^2}{s} \tag{41}$$

$$\lim_{t\to\infty} Y(t) = \lim_{s\to 0}\left(sY(s)\right) = \lim_{s\to 0}\left(sG_{Y\delta 0}(s)\delta(s)\right) = \lim_{s\to 0}\left(s\frac{G_{\Omega\delta 0}V\left(T_1^2s^2+2\xi_1T_1s+1\right)}{s^2\left(T_0^2s^2+2\xi_0T_0s+1\right)}\delta_0\frac{\left(1-e^{-sT}\right)^2}{s}\right) = T^2G_{\Omega\delta 0}V\delta_0 = Y_0 \tag{42}$$

$$\lim_{t\to\infty} \psi(t) = \lim_{s\to 0}\left(s\psi(s)\right) = \lim_{s\to 0}\left(sG_{\Omega\delta}(s)\delta(s)\right) = \lim_{s\to 0}\left(s\frac{G_{\Omega\delta 0}\left(T_{\Omega\delta}s+1\right)}{s\left(T_0^2s^2+2\xi_0T_0s+1\right)}\delta_0\frac{\left(1-e^{-sT}\right)^2}{s}\right) = 0 \tag{43}$$

The meaning of the above results is that vehicle steer through a sharp pull of the steering wheel in one direction, followed by another pull in the opposite direction, causes the vehicle to change the lane of travel. This conclusion is also supported by observations of real vehicle behaviour. For the development of the controller, it is crucial to note that according to the reference model on the new lane the vehicle will move with zero yaw angle. Of course, due to the presence of disturbances and imperfections of the reference model after reaching the steady state the vehicle may be moving along a straight path with nonzero yaw angle. Eliminating that error will be the subject of the corrective action of the controller.
To achieve the intended lane change represented by the lateral distance Y_0, it is necessary to appropriately choose the time duration T of the control impulse (square relationship) and its value δ_0 (linear relationship). In this process it is necessary to take into account the value of the amplification parameter $G_{\Omega\delta 0}$ and vehicle velocity V.
The reference model proposed above can be used directly in the control of the trajectory of the vehicle centre of mass (the first phase of the manoeuvre) as well as in the vehicle yaw angle control (the second phase).

Control of the Lateral Displacement of the Vehicle Mass Centre

According to the proposed control approach, in the first phase of the process, the control of the steering wheel rotation angle will be executed in the open loop – on the basis of the reference model, taking into account signal limitations and minimization of the manoeuvre duration time. At the same time, the corrective action will be carried out based on the principles of automatic regulation. Accuracy of this operation should be ensured mostly by the quality of the identified reference model. The diagram of the control system for this phase is shown in Fig. 4.

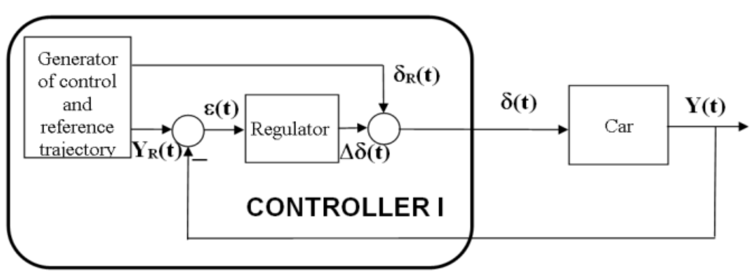


Figure 4. Block diagram of the automatic control system in Phase I.

Generator of steering wheel angle and reference trajectory Reference time profiles $\delta_R(t)$ and $Y_R(t)$ are generated.

The reference time profile of vehicle lateral displacement $Y_R(t)$ that ensures obstacle avoidance is determined by generation of the control input in the form presented in Eq. (40), with parameters δ_0 and T chosen such that the conditions $|\ddot{y}(t)| \le \ddot{y}_{dop}$ and $|\dot{\psi}(t)| \le \dot{\psi}_{dop}$ are satisfied, and the steady state is achieved within the time not exceeding the allowed value resulting from current vehicle velocity and the distance from the obstacle. For the selection of δ_0 and T, trajectory optimization can be used, for example with the objective to achieve the shortest time to reach the steady state. Such selection and representing the input function parameters in the tabular form are to be carried out off-line on the basis of simulations, for a broad range of design and operational parameters present in the initial reference model. The simplified reference model can facilitate that process.

The regulator The regulator can be developed on the basis of the classical control theory, or with the use of the optimal control theory applied to the linear-quadratic problem. As known, regulators determined from the solution of linear-quadratic problems are usually very effective and relatively simple for implementation. Such an approach is presented below.

The linear-quadratic problem is formulated as follows. For the model in the state-space matrix form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{44}$$

with initial conditions $\mathbf{x}(0) = \mathbf{0}$ find the control vector $\mathbf{u}(t)$ that minimizes the functional:

$$Q = \int_0^\infty \left(\mathbf{x}(t)^T \mathbf{P}\mathbf{x}(t) + \mathbf{u}(t)\mathbf{R}\mathbf{u}(t)\right)dt \ . \tag{45}$$

with **P** and **R** – positively defined weight matrices.

According to Kalman theorem [1] the solution of the linear-quadratic problem is:

$$\hat{\mathbf{u}}(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}\mathbf{x}(t) \tag{46}$$

where **K** –symmetric matrix satisfying Ricatti equation: $-\mathbf{KA} - \mathbf{A}^T\mathbf{K} + \mathbf{KBRB}^T\mathbf{K} = \mathbf{P}$ (47)

The meaning of this solution is that the control vector is computed in the closed loop system, with feedback parameters depending on the model and the objective weight matrices, and determined as the solution of the nonlinear algebraic Ricatti equation.

In order to apply the theory of linear-quadratic systems it is necessary to formulate the linear mathematical model of the object in the state-space form. The control performance index should be presented in the form of integral functional with quadratic forms. Note that the earlier determined transfer function $G_{Y\delta}(s)$ describes also the dynamics for the perturbed states:

$\Delta\delta = \delta_R(t) - \delta(t)$ and $\Delta Y = Y_R(t) - Y(t) = \varepsilon(t)$ (tracking error), and therefore one also has:

$$\Delta Y(s) = G_{Y\delta}(s)\Delta\delta(s) = \frac{G_{\Omega\delta 0}V\left(T_1^2 s^2 + 2\xi_1 T_1 s + 1\right)}{s^2\left(T_0^2 s^2 + 2\xi_0 T_0 s + 1\right)}\Delta\delta(s) \tag{48}$$

For the purpose of illustrating the method by analytical means the simplified reference model will be used together with simplified form of the transfer function:

$$G_{Y\delta}(s) = \frac{\Delta Y(s)}{\Delta\delta(s)} = \frac{G_{\Omega\delta 0}V\left(T_1^2 s^2 + 2\xi_1 T_1 s + 1\right)}{s^2\left(T_0^2 s^2 + 2\xi_0 T_0 s + 1\right)} \approx \frac{G_{\Omega\delta 0}V}{s^2} \tag{49}$$

The model that corresponds to such transfer function has the following state-space form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t) \tag{50}$$

where

$$x_1(t) = \Delta Y(t) \quad x_2(t) = \Delta\dot{Y}(t) \quad u(t) = G_{\Omega\delta 0}V\Delta\delta(t) \tag{51}$$

The control performance index can be defined as:

$$Q = \int_0^\infty \left(p_{11}x_1^2(t) + p_{22}x_2^2(t) + ru^2(t)\right)dt \text{ which means}$$

$$Q = \int_0^\infty \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} p_{11} & 0 \\ 0 & p_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + uru(t)\right)dt \tag{52}$$

In the typical notation of the linear-quadratic problem one gets:

$$\mathbf{A}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}0\\ 1\end{bmatrix}\quad \mathbf{P}=\begin{bmatrix}p_{11} & 0\\ 0 & p_{12}\end{bmatrix}\quad \mathbf{R}=r \tag{53}$$

The linear-quadratic problem has a solution according to Eqs. (46) and (47). For the considered case:

$$\mathbf{R}^{-1}=\frac{1}{r} \tag{54}$$

$$\mathbf{K}=\begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}=\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}$$

The symmetric matrix **K** satisfies the equation:

$$-\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}-\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}^T\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}+\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}r\begin{bmatrix}0\\ 1\end{bmatrix}^T\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}=\begin{bmatrix}p_{11} & 0\\ 0 & p_{22}\end{bmatrix} \tag{55}$$

Carrying matrix operations in Eq. (55) leads to the equation:

$$\begin{bmatrix}rK_{12}^2 & -K_{11}+rK_{12}K_{22}\\ -K_{11}+rK_{12}K_{22} & -2K_{12}+rK_{22}^2\end{bmatrix}=\begin{bmatrix}p_{11} & 0\\ 0 & p_{22}\end{bmatrix} \tag{56}$$

Considering stability requirements one finally gets:

$$K_{12}=\sqrt{\frac{p_{11}}{r}},\qquad K_{22}=\sqrt{\frac{p_{22}+2\sqrt{\frac{p_{11}}{r}}}{r}},\qquad K_{11}=\sqrt{p_{11}\left(p_{22}+2\sqrt{\frac{p_{11}}{r}}\right)} \tag{57}$$

Substituting calculated coefficients into Eq. (46) gives:

$$\hat{u}(t)=-\frac{1}{r}\begin{bmatrix}0\\ 1\end{bmatrix}^T\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}=-\frac{1}{r}\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}=-\frac{1}{r}\begin{bmatrix}K_{12} & K_{22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}=-\frac{1}{r}\left(K_{12}x_1(t)+K_{22}x_2(t)\right) \tag{58}$$

Hence

$$\hat{u}(t)=-\frac{1}{r}\left(\sqrt{\frac{p_{11}}{r}}x_1(t)+\sqrt{p_{11}\left(p_{22}+2\sqrt{\frac{p_{11}}{r}}\right)}x_2(t)\right) \tag{59}$$

$$\Delta\hat{\delta}(t)=-\frac{1}{G_{\Omega\delta 0}Vr}\left(\sqrt{\frac{p_{11}}{r}}\Delta Y(t)+\sqrt{p_{11}\left(p_{22}+2\sqrt{\frac{p_{11}}{r}}\right)}\Delta\dot{Y}(t)\right) \tag{60}$$

In the operator domain:

$$\Delta\hat{\delta}(s)=-\frac{1}{G_{\Omega\delta 0}Vr}\left(\sqrt{\frac{p_{11}}{r}}+s\sqrt{p_{11}\left(p_{22}+2\sqrt{\frac{p_{11}}{r}}\right)}\right)\Delta Y(s) \tag{61}$$

The transfer function of the proportional-plus-derivative regulator (PD):

$$G_{\Delta\delta\Delta Y}(s)=\frac{\Delta\delta(s)}{-\Delta Y(s)}=\frac{1}{G_{\Omega\delta 0}Vr}\left(\sqrt{\frac{p_{11}}{r}}+s\sqrt{p_{11}\left(p_{22}+2\sqrt{\frac{p_{11}}{r}}\right)}\right) \tag{62}$$

With the use of the vehicle lateral acceleration the controller becomes the regulator with integration.

Stabilization of the Yaw Angle of the Vehicle

Following the earlier described concept of vehicle control, in the second phase of the manoeuvre the control of the steering wheel rotation will be carried out as in the regulation system. Considering possible unification of the control system, it is useful to set the block structure of the controller in the form similar to the first phase controller. It is shown in Fig. 5.

In this case the generator of reference signals has a trivial form:

$$\psi_R(t)=0\qquad \delta_R(t)=0 \tag{63, 64}$$

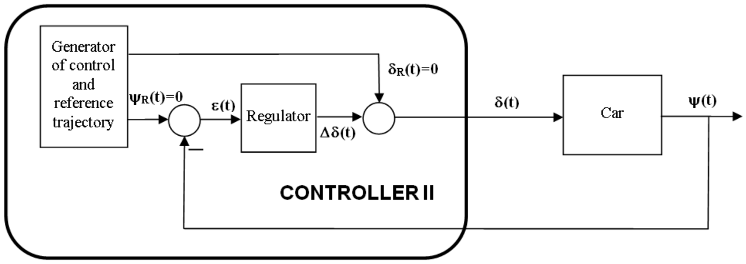


Figure 5. Block diagram of the automatic control system in Phase II.

The regulator adjusting the steering wheel rotation angle can be developed in the way similar to Phase I. Note that in this case the transfer function $G_{\Omega\delta}(s)$, determined from the initial reference model, can be also used for the description of the dynamics with angular perturbations:

$\Delta\delta(t) = \delta_R(t) - \delta(t)$ and $\Delta\psi = 0 - \psi(t) = \varepsilon(t)$ (regulation error), and thus one gets:

$$\Delta\psi(s) = \frac{1}{s} G_{\Omega\delta}(s)\Delta\delta(s) = \frac{G_{\Omega\delta 0}(T_{\Omega\delta}s + 1)}{s(T_0^2 s^2 + 2\xi_0 T_0 s + 1)}\Delta\delta(s) \tag{65}$$

Subsequent development follows an analogous path to the already presented and is omitted in this paper.

Note that due to identical structures of the controllers for the first and second phases of the manoeuvre, an universal controller device can be used, with a switchable algorithms and changing reference and feedback signals.

The developed algorithms of the controller, generating the reference signals and error feedback terms, constitute the basis of the active steering system controller. In the simplest approach, the steering wheel rotation angle $\delta_H(t)$ can be treated as a scaled version (through the steering gear ratio) of the steer angle of the wheels $\delta(t)$. In the more comprehensive solutions, additional corrective terms can be added in order to take into account the dynamics and nonlinearities of the real steering mechanism.

SIMULATION RESULTS

Verification and validation of the proposed control approach was carried out through numerical simulations in which a comprehensive model of vehicle dynamics was used. That model was treated as a “real” vehicle. The model of a commercial truck [5,6] was adapted for that purpose and used in simulations. It represents a complex, 3D model of a two-axis commercial truck of medium load capacity, having twenty degrees of freedom, and built on the basis of studies and observations of the real vehicle (STAR 1142). The active steering system included in the model takes into account its geometry, kinematics, dynamics as well as elastic and damping properties. The tire model proposed by Dugoff, Fancher, Segel [3], completed with recommendations resulting from the research conducted under the guidance of Mitschke [7], was used for the description of interaction between vehicle tires and roadway surface. An important advantage of this tire model is that in spite of relatively simple mathematical formulation it allows for an easy introduction of vehicle parameters (traction coefficient, velocity, radial loads) and makes it possible to simulate vehicle motion in the full skid condition.

The model of the truck was subjected to a broad and thorough experimental verification [5]. The results obtained during tests of the real vehicle were used for the experimental verification of the model. Typical manoeuvres included driving along a circular path in steady conditions, quick turn of the steering wheel while driving straight ahead and braking while driving along a straight road and braking while turning. In order to determine the parameters of the tire model, thorough experimental tests of dynamic characteristics of vehicle tires were carried out on a drum dyno and with a dynamometer trailer.

Effectiveness of the control approach presented in this paper has been evaluated through simulations that involved avoiding suddenly appearing obstacle through single lane change on a shortest distance possible. During numerical experiments it was necessary to tune the values of parameters p_{11}, p_{22} and r (Eq. 60). Adjusting those values was done by trial and error. A number of obstacle avoidance manoeuvres with different settings were carried out. The parameters that were changing in the consecutive tests included the initial velocity of the vehicle (in the range of V=40-80km/h), the friction coefficient between the tires and the road surface (in the range of μ=0.1-0.5) and the weight of the load in the cargo section of the truck (not loaded, partly loaded and fully loaded). The objective was to find the settings that would allow successful completion of the obstacle avoidance manoeuvre in all trials that were conducted.

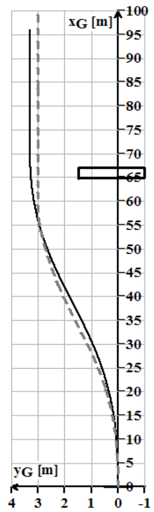
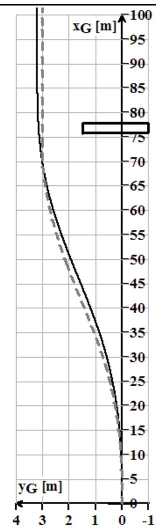
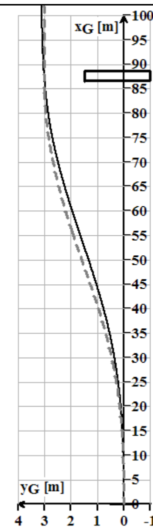
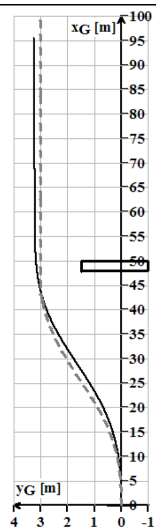
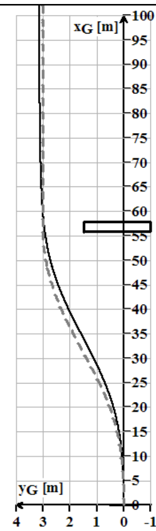
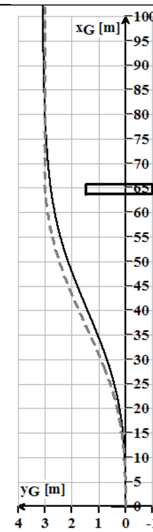
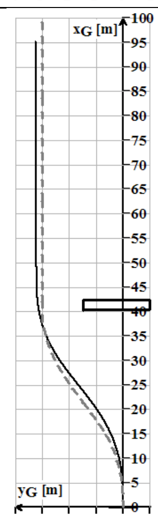
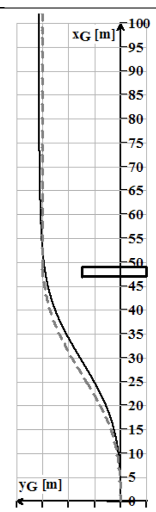
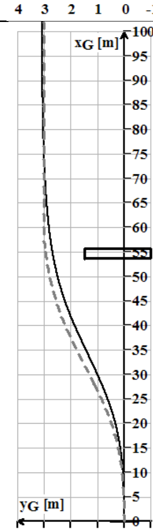
Item	V=60km/h	V=70km/h	V=80km/h
μ=0.1			
μ=0.2			
μ=0.3			

Figure 6. The results of the simulation studies – the assigned (broken line) and realized (solid line) trajectory of the car's centre of mass.

The obstacle avoidance manoeuvre, according to the earlier described approach, was implemented in two phases. The controller shown in Fig. 4 was used in the first phase. In that phase, the desired trajectories $Y_R(t)$ of the vehicle centre of mass had been generated using the bicycle model of the vehicle and Eq. (40). The time profiles of the steering wheel rotation angle $\delta_{HR}(t)$ had been determined using Eq. (40) and the mean value of the steering system ratio. For each test, the values of variables T and δ_0 (Fig. 3) were determined taking into account the limit value of the radius of the circular path on which the vehicle could move without the loss of lateral traction. The lateral displacement Y_0=3 m of vehicle's centre of mass to the target lane was used. The controller shown in Fig. 5 was used in the second phase of the manoeuvre. In that phase, the vehicle yaw angle was set to zero ($\psi(t)=0$) and the steering wheel rotation angle was assigned as $\delta_{HR}=0$. In both phases, the instantaneous value of the steering wheel rotation angle $\delta_H(t)$ taken as the sum of the assigned value $\delta_{HR}(t)$ and the value $\Delta\delta_H(t)$ computed by the regulator was limited by the allowable values of vehicle velocity and the values of steering wheel angular acceleration. In all simulations that were carried out the controller II (Fig. 5) was taking over the control of the steering wheel rotation angle after the time period of t_1=1.5T (Fig. 3).
The results of simulations have shown that in all tests that were carried out the obstacle avoidance manoeuvre was accomplished successfully. A portion of simulation results is presented in Fig. 6. They were obtained for the fully loaded truck, with the low position of the centre of mass, on slippery pavements (friction coefficient of μ =0.1-0.3), and with vehicle velocities of 60-80km/h.
The automatically controlled vehicle was able to avoid the obstacle without losing directional stability, even though the conditions of vehicle operation were changing in a broad range. Therefore, it can be stated that the proposed concept of control and the developed regulators proved to be insensitive to varying road conditions, and with that, the control approach appeared to be effective in realization of the lane change manoeuvre on the shortest path possible.

CONCLUSIONS AND CLOSING REMARKS

The adopted reference model of the lateral dynamics of the vehicle in its initial and simplified versions seems to provide an accepted basis for the development of controllers that automate the lane change manoeuvre. The proposed method of the decomposition of the control leads to two identical structures of the controller for both phases of the process. The results of simulations showing the use of such controller for the control of the commercial truck prove the correct direction of the research.
The method of automatic execution of the lane change manoeuvre presented in the paper can offer an attractive alternative for vehicle control engineers and researchers working in the fields of automatic steering systems and vehicle active safety systems. The results are especially important because they illustrate the application of the methodology for the case of a commercial truck.

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