Probabilistic Prediction based Automated Driving Control in Urban Traffic Situation

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ABSTRACT
This paper represents an automated driving control algorithm in urban traffic situation. In order to achieve a development of a highly automated driving control algorithm in urban environments, the research issues can be classified into two things. One of the issues is to determine a safe driving envelope with the consideration of probable risks and the other is to achieve robustness of control performance under disturbances and model uncertainties. While human drivers maneuver a vehicle, they determine appropriate steering angle and acceleration based on the predictable trajectories of the surrounding vehicles. Therefore, not only current states of surrounding vehicles but also predictable behaviors of surrounding vehicles and potential obstacles should be considered in designing an automated driving control algorithm. In order to analyze the probabilistic behaviors of surrounding vehicles, we collected driving data on a real road. Then, in order to guarantee safety to the possible change of traffic situation surrounding the subject vehicle during a finite time-horizon, the safe driving envelope which describes the safe driving condition over a finite time horizon is defined in consideration of probabilistic prediction of future positions of surrounding vehicles and potential obstacles. Since an automated driving control algorithm is required to operate in a wide operating region and limit the set of permissible states and inputs, a model predictive control (MPC) approach has been used widely in designing an automated driving control algorithm. MPC approach uses a dynamic model of the vehicle to predict the future states of the system and determines optimal control sequences at each time step to minimize a performance index while satisfying constraints based on the predicted future states. Since the solving nonlinear optimization problem has computational burden, we design an architecture which decides a desired steering angle and longitudinal acceleration parallel to reduce the computational load. For the guarantee of the robustness of control performance, a robust invariant set is used to ensure robust satisfaction of vehicle states and constraints against disturbances and model uncertainties. The effectiveness of the proposed control algorithm is evaluated by comparing between human driver data and proposed algorithm.

I. Introduction

Recently, the interest of automotive industry changes from the passive safety system to the active safety system and, by extension, automated driving system due to advances in sensing technologies. For example, active safety applications, such as vehicle stability control (VSC), adaptive cruise control (ACC), lane keeping assistance (LKA) and lane change assistance (LCA) system, have been extensively researched [1]. In order to enhance safety and achieve zero fatalities, many researches have been undertaken to integrate individual active safety systems for the development of an automated driving system [2].

In developing an automated driving system which is required to operate in a wide operating region and limit the set of permissible states and inputs, MPC approach has been used widely because of its capability to handle system constraints in a systematic way [3], [4]. MPC approach uses a dynamic model of the plant to predict the future states of the system and determines optimal control sequences at each time step to minimize a performance index while satisfying constraints based on the predicted future states [5]. The first term of this optimal control sequences is applied to the system. At next time step, new optimal control sequences is calculated over a shifted prediction horizon. In [6], Falcone et al. present a MPC based active steering controller for tracking the desired trajectory as close as possible while satisfying various constraints. In this research, it is assumed that the desired trajectory over a finite horizon is known. Erlien et al. use a safe driving envelope which means a safe region of states in which the system should be constrained [7]. In this research, the safe driving envelope consists of a stable handling envelope to ensure vehicle stability and an environmental envelope to constrain the position states for the collision avoidance. The environmental envelope is defined based on the current states of surrounding environment of the subject vehicle. In order to compensate the effect on the control performance by model uncertainties and exogenous disturbances, robust MPC approach which adds a linear feedback control input to the nominal control inputs based on the analysis of robust invariant sets have been introduced and used to design an autonomous control algorithm [8].

In order to develop a highly automated driving system, the research issues can be classified into two things. One of the issues is to enhance safety under the possible change of the behaviors of neighboring vehicles in the future. Human drivers maneuver the vehicle predicting possible surrounding vehicle's trajectories. Therefore, not only current states of surrounding environment of the subject vehicle but also predicted behaviors of surrounding environment should be considered to control the vehicle autonomously [9]. Furthermore, since probable behaviors of surrounding vehicles should be considered to prevent a potential collision accident in the future, a probabilistic prediction is required [10]. The other issue in designing an automated driving system is to achieve robustness of control performance under disturbances and model uncertainties due to inaccurate or time varying parameters [6].

In this research, we focus on designing an automated control algorithm which handles probable risky situations due to the possible change of traffic situation surrounding the subject vehicle while satisfying a robust control performance with respect to model parameter uncertainties and exogenous disturbances. In order to enhance safety with respect to the potential behaviors of surrounding vehicles, a safe driving envelope which describes the safe driving condition over a finite time horizon is defined in consideration of probabilistic prediction of future states of surrounding environment. Then MPC problem is formulated to determine the desired steering angle and desired longitudinal acceleration while maintaining the subject vehicle into the safe driving envelope. A tube-based robust MPC approach is used to guarantee robust performance under model uncertainties and exogenous disturbances.

This paper is structured as follows: The overall architecture of the proposed automated driving control algorithm is described in Section II. In Section III, the lateral dynamics model for the determination of the desired steering angle and longitudinal dynamics model for the determination of the desired longitudinal acceleration are derived briefly. In Section IV, probabilistic prediction of surrounding vehicle behaviors and the description of the safe driving envelope is described briefly. Then the controller is designed based on robust MPC approach in Section V. Section VI shows the vehicle test results for the evaluation of the performance of the proposed algorithm. Then the contribution of this research and introduction of future works are summarized in Section VII.

II. Overall Architecture

The overall architecture of the proposed automated driving control algorithm is shown in Fig. 1. In the integrated perception layer, the information which is required to determine the desired driving mode and safe driving envelope is refined using measurements from various sensors. In order to assess the driving situation precisely, states of the subject vehicle and surrounding vehicles should be estimated from various measurements via exterior sensors, such as vision and radar sensors. Then, the probable behaviors of the surrounding vehicles over a finite prediction horizon are predicted using the information of current states of surrounding vehicles. Using the estimated states of the subject vehicle and the ranges of probable behaviors of the surrounding vehicles over a finite prediction horizon, a desired motion or desired driving mode of the subject vehicle is determined in the risk management layer. Since the goal of the automated driving control algorithm proposed in this paper is to control the vehicle autonomously on the road, the required driving mode is classified into lane keeping and lane change mode. The desired driving mode is determined with the consideration of not only current states of traffic situation surrounding the subject vehicle but also predictable situations among the potential changes of traffic situation surrounding the subject vehicle. Then the safe driving envelope is determined based on the desired driving mode. Then the controller is designed to determine the desired steering angle and the desired longitudinal acceleration separately while satisfying reliability. Using robust MPC approach, the desired control inputs are determined to improve safety and ride comfort while satisfying constraints of states and inputs.

III. Vehicle dynamics model

In order to obtain the desired control inputs separately based on MPC approach, the lateral dynamics model and longitudinal dynamics model should be derived. In this research, the lateral dynamics model is designed by combining the bicycle model and error dynamics with respect to a road. Furthermore, the longitudinal dynamics model is designed by integrating the inter-vehicle dynamics and longitudinal actuator's dynamics.

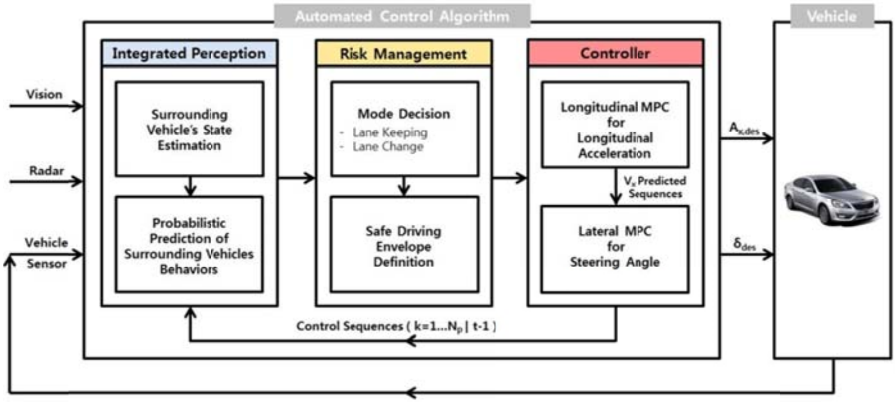


Fig. 1 Overall architecture of the proposed automated driving control algorithm

A. Lateral dynamics model

A classic bicycle model is usually used to design a lateral control law [3]. However, since an automated driving system should operate in a wide operating region, a classic bicycle model which assumes small slip angles of tires could not be suitable as a predictive model. On the other hand, if we use a nonlinear tire model to build a dynamic model, a nonlinear optimization problem should be solved at each time step. However, a computational burden to solve a nonlinear MPC problem is a critical barrier for its implementation [6]. In order to cope with this drawback, we apply a saturated linear tire model to reflect a tire saturation characteristic [11]. Then the bicycle model could be modified as follows:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \delta_f \tag{1}$$

$$\begin{aligned} & a_{11} = -\frac{2k_{\alpha f}C_f + 2k_{\alpha r}C_r}{mv_x}, a_{12} = -1 + \frac{-2k_{\alpha f}C_f l_f + 2k_{\alpha r}C_r l_r}{mv_x^2}, \\ & a_{21} = \frac{-2k_{\alpha f}C_f l_f + 2k_{\alpha r}C_r l_r}{I_z}, a_{22} = -\frac{2k_{\alpha f}C_f l_f^2 + 2k_{\alpha r}C_r l_r^2}{I_z v_x}, \\ & b_1 = \frac{2k_{\alpha f}C_f}{mv_x}, b_2 = \frac{2k_{\alpha f}C_f l_f}{I_z} \end{aligned} \tag{2}$$

where, $k_{\alpha f}$ and $k_{\alpha r}$ are the cornering stiffness adjustment coefficients to reflect a tire saturation characteristic. These adjustment coefficients are assumed to be known exactly in this paper.

In order to control the vehicle in the lateral direction, the modified bicycle model is combined with the error dynamics which describes error with respect to a road. Therefore, the complete model used to design a MPC controller is defined as shown in (3) and a diagram of the vehicle model is depicted in Fig. 2.

$$\dot{x}_{lat} = A_{lat}x_{lat} + B_{lat}u_{lat} + F_{\rho,lat}\rho_{ref} \tag{3}$$

$$A_{lat} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ v_x & 0 & v_x & 0 \end{bmatrix},\ B_{lat} = \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix},\ F_{\rho,lat} = \begin{bmatrix} 0 \\ 0 \\ -v_x \\ 0 \end{bmatrix} \tag{4}$$

where, the state vector is $x_{lat} = \begin{bmatrix} \beta & \gamma & e_\psi & e_y \end{bmatrix}^T$, the control input is $u_{lat} = \delta_{f,des}$, e_ψ is the orientation error of the vehicle with respect to the road, e_y denotes the lateral offset with respect to the center line of the lane, and ρ_{ref} is the road curvature.

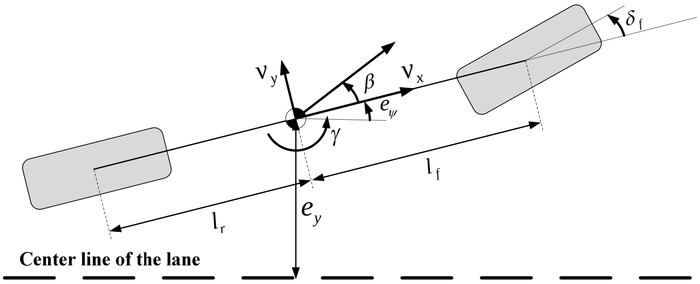


Fig. 2 Diagram of lateral dynamics model

In order to solve a receding horizon optimization problem, the continuous differential equation (3) should be discretized. (1) can be converted as follows:

$$x_{lat}(k+1|t) = A_{lat,d}(k|t)x_{lat}(k|t) + B_{lat,d}(k|t)u(k|t) + F_{\rho,lat,d}(k|t)\rho_{ref}(t) \tag{5}$$

$$A_{lat,d} = e^{A_{lat}T_s},\ B_{lat,d} = \left(\int_0^{T_s} e^{A_{lat}\tau} d\tau\right)B_{lat},\ F_{\rho,lat,d} = \left(\int_0^{T_s} e^{A_{lat}\tau} d\tau\right)F_{\rho,lat} \tag{6}$$

where, T_s is the sampling time. The system matrices of the lateral dynamics model, such as $A_{lat,d}(k|t)$, $B_{lat,d}(k|t)$, and $F_{\rho,lat,d}(k|t)$, are obtained using the predicted sequences of the longitudinal velocity during a finite time-horizon.

B. Longitudinal dynamics model

In designing a longitudinal dynamics model of the subject vehicle, an actuator delay between the desired longitudinal acceleration and the response of the actual longitudinal acceleration is considered as follows [11]:

$$a_x = \frac{1}{\tau_{ax}s+1}a_{x,des} \tag{7}$$

where, τ_{ax} is a time-constant chosen as 0.4 sec based on the analysis of the vehicle test platform.

In this research, two variables, such as distance error Δd and relative speed Δv_x, are used to define the inter-vehicle dynamics.

$$\begin{aligned} &\Delta d = C_x - C_{x,des},\quad C_{x,des} = \tau_h \cdot v_x + C_{x,safe} \\ &\Delta v_x = v_{x,\text{target}} - v_x \end{aligned} \tag{8}$$

where, C_x and $C_{x,des}$ are the actual clearance and desired clearance between the subject vehicle and the target vehicle respectively, τ_h indicates the time gap, $C_{x,safe}$ is the minimum safety longitudinal clearance and $v_{x,\text{target}}$ is the longitudinal velocity of the target vehicle. In this research, in order to embrace driving characteristics of all of the drivers, the time gap, τ_h, is chosen as 1.36 sec which is the mean value of time gap for collected driving data in steady-state following situation [1]. Furthermore the minimum safety longitudinal clearance, $C_{x,safe}$, is chosen as 2 meters which is identical with the mean value of the clearance at the zero speed for all of the drivers [1]. The method how to select the target vehicle among the surrounding vehicle would be described in Section IV.

The derivative of the equation (8) could be derived as shown in (9)

$$\begin{aligned} &\Delta\dot{d} = \Delta v_x - \tau_h \cdot a_x \\ &\Delta\dot{v}_x = a_{x,\text{target}} - a_x \end{aligned} \tag{9}$$

Combining equation (7) and equation (9), the longitudinal dynamics model could be described as follows:

$$\dot{x}_{long} = A_{long}x_{long} + B_{long}u_{long} + F_{long}a_{x,\text{target}} \tag{10}$$

$$A_{long} = \begin{bmatrix} 0 & 1 & -\tau_h \\ 0 & 0 & -1 \\ 0 & 0 & -1/\tau_{ax} \end{bmatrix}, B_{long} = \begin{bmatrix} 0 \\ 0 \\ 1/\tau_{ax} \end{bmatrix}, F_{long} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{11}$$

where, the state vector is $x_{long} = [\Delta d \quad \Delta v_x \quad a_x]^T$ and the control input is $u_{long} = a_{x,des}$.

As similar as the lateral dynamics model, the discretization of the continuous state equation (10) is conducted through the ZOH method as follows:

$$x_{long}(k+1) = A_{long,d}x_{long}(k) + B_{long,d}u_{long}(k) + F_{long,d}a_{x,\text{target}}(k) \tag{12}$$

$$A_{long,d} = e^{A_{long}T_s},\ B_{long,d} = \left(\int_0^{T_s} e^{A_{long}\tau}d\tau\right)B_{long},\ F_{\rho,long,d} = \left(\int_0^{T_s} e^{A_{long}\tau}d\tau\right)F_{\rho,long} \tag{13}$$

IV. Safe driving Envelope

Generally, human drivers monitor surrounding environment and predict the future states of surrounding environment based on the current states of that. Then drivers estimate the threat level of possible actions and decide the maneuver of the subject vehicle in consideration of the predicted states of surrounding vehicles during a finite time-horizon. Therefore, in order to develop a highly automated driving system, a safe driving envelope which indicates the drivable boundaries for safe driving over a finite prediction horizon should be determined with the consideration of not only current states of traffic situation surrounding the subject vehicle but also probable future states of that simultaneously [9]. Considering probable future states of surrounding vehicles, it could be expected that the automated driving control algorithm could handle probable risky situation during a finite time-horizon and enhance safety. Furthermore, if we define the safe driving envelope based on the probabilistic prediction, it is expected that an automated driving control algorithm which reflects human driver's driving characteristics with an acceptable ride comfort could be developed. Firstly, the method of the probabilistic prediction method is presented in Section III-A. Then the determination of the desired driving mode and the safe driving envelope is represented in Section III-B.

A. Probabilistic prediction of surrounding vehicle's behavior

One of common approach to predict the future states of traffic situation surrounding the subject vehicles is a deterministic prediction which assumes that the surrounding vehicles maintain its current movement during a finite time horizon. Since this approach ignores the probability of all possible movements of surrounding vehicles, this could cause incorrect interpretation of the current driving situation.

In order to compensate the shortcomings of the deterministic prediction of the behaviors of surrounding vehicles, the possible behaviors of surrounding vehicles are predicted and the risky behaviors among the possible behaviors of other vehicles surrounding the subject vehicle are considered in determining the safe driving envelope.

For the prediction of the reasonable and realistic behaviors of surrounding vehicles, the interaction between vehicles and the restriction on surrounding vehicle's maneuver due to the road geometry should be considered [12]. Moreover, it is assumed that drivers of the surrounding vehicles obey general traffic rules [13]. It means that the surrounding vehicle's behavior is assumed to keep the lane or change one lane at a time, not two or more lanes at a time. If one of surrounding vehicles changes the lane, then that vehicle is assumed to keep the relevant lane in the far-off future. Furthermore the violation of the centerline of surrounding vehicles is prohibited.

In predicting reasonable ranges of the future states of surrounding vehicles, driving data are collected on test track and real road to analyze the probabilistic movement characteristics of the vehicle [14]. For the implementation of these assumptions, a path-following model is designed while interacting with a vehicle state predictor during one cycle of the prediction process. In the vehicle state predictor, the vehicle's probable position and its error covariance over a finite time horizon are predicted by *Extended Kalman Filter* using the desired yaw rate obtained by the path-following model as the virtual measurement.

Fig. 3 depicts the overall architecture of probabilistic prediction of surrounding vehicles. Using measurements from the various sensors, such as vehicle sensor, radar and vision sensor, the range of the predicted states with corresponding uncertainty is determined as shown in Fig. 3. p_x is the longitudinal position of the vehicle, p_y is the lateral position of the vehicle, N_p denotes the prediction horizon, and subscript '*j*' means the *j*-th objects. In predicting the position of the surrounding vehicle, it is assumed that the size of the object is equivalent to the subject vehicle. The ellipse in Fig. 3 indicates the predicted probable range of the center gravity of the vehicle at the prediction time. A detailed description on the computational procedures to predict the probabilistic range of future states during a finite time horizon is described concretely in [10], [15].

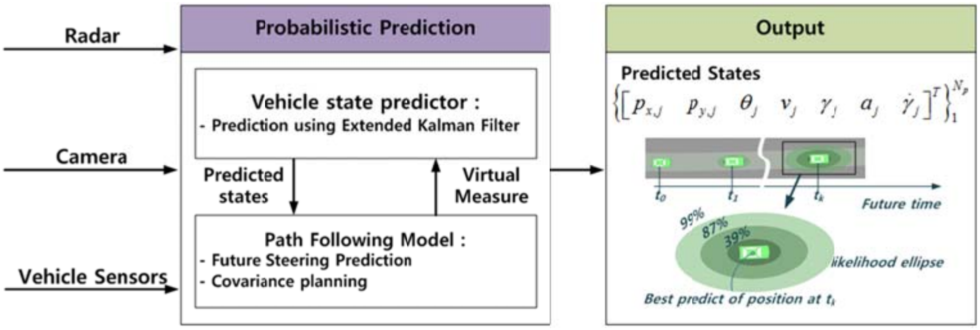


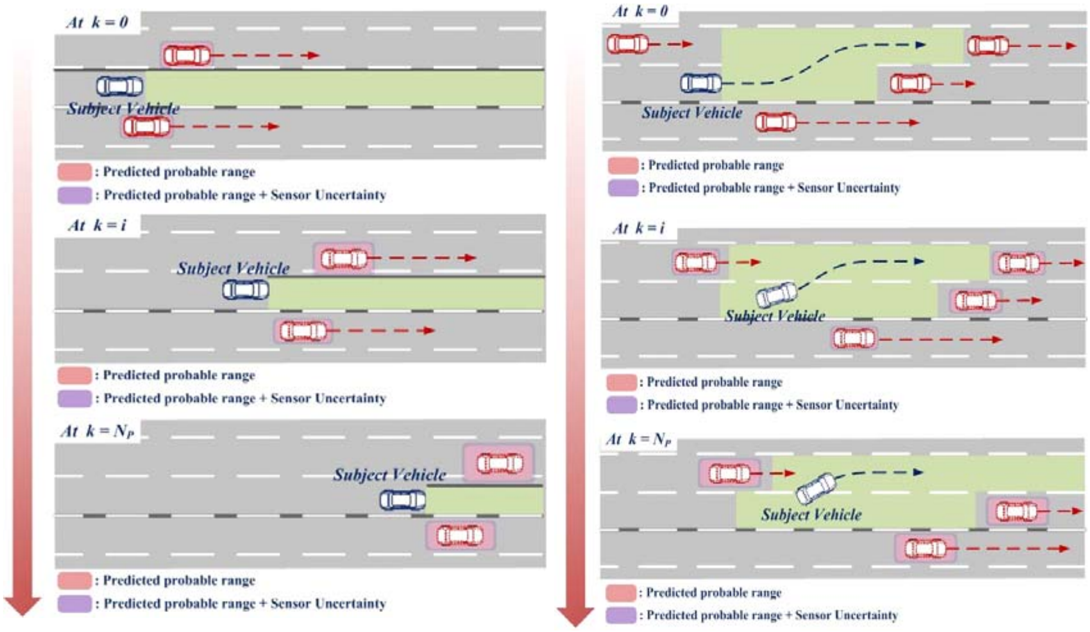
Fig. 3 Overall architecture of probabilistic prediction of surrounding vehicle's behavior

B. Driving mode and Environmental envelope decision

For the determination of the environmental envelope to improve safety, first of all, a potential risky situation should be considered. The risky situation among the probable behaviors of the surrounding vehicles could be classified roughly into three types. Firstly, if the preceding vehicle in the originating lane of the subject vehicle decelerates abruptly, then the potential risk of collision between the preceding vehicle and the subject vehicle would increase. Secondly, if the approaching vehicle in the adjacent lane accelerates during a lane change maneuver of the subject vehicle, then the collision between the approaching vehicle in the adjacent lane and the subject vehicle could be expected. Thirdly, there could be a potential risk of collision due to a sudden cut-in vehicle. Therefore, for the enhancement of safety, not only current states of surrounding environment of the subject vehicle but also these risky behaviors of the surrounding vehicles over a finite prediction horizon should be considered in determining the environmental envelope to improve safety.

Since the environmental envelope should be defined based on the desired motion, we should determine the desired motion or desired driving mode of the subject vehicle before the decision of the environmental envelope. The required driving mode could be approximately classified into lane keeping and lane change mode on an auto road. If there is no preceding vehicle in the originating lane which has a potential risk of collision during a finite prediction horizon, the desired driving mode could be determined as a lane keeping mode. In this case, the environmental envelope is determined to keep the originating lane while maintaining safety with respect to the surrounding vehicle. If the longitudinal or lateral clearances expected at the prediction time step k between the subject vehicle and surrounding vehicle are larger than predefined threshold value, then the collision risk is low and the environmental envelope for e_y is determined to prevent a lane departure. On the other hand, if the longitudinal or lateral clearances at the prediction time step k are expected to be smaller than thresholds, then the collision risk is high. Therefore the environmental envelope for e_y is determined to keep the originating lane while evading the approaching vehicle in the adjacent lane. The decision process of the environmental envelope for a lane keeping mode and the environmental envelope to keep the originating lane while maintaining safety with respect to the surrounding vehicles are described in Fig. 4-(a). In Fig. 4-(a), the pink rectangle indicates the region of the possible behavior of surrounding vehicles and the violet rectangle indicates the region of the possible behavior of surrounding vehicles with the consideration of sensor uncertainty.

On the other hand, there could be a preceding vehicle in the originating lane which has a collision risk during a finite prediction horizon or one of the surrounding vehicles in the adjacent lane is expected to change the lane into the originating lane of the subject vehicle during the prediction time horizon. In this case, the lane change of the subject vehicle from the originating lane to the adjacent lane might be required. Then the feasibility of the lane change and safety after the lane change should be considered. If there is no vehicle is in the adjacent lane when the lane change of the subject vehicle is required, then the lane change could be permitted. Otherwise, we should investigate the minimum longitudinal clearance between the subject vehicle and the vehicle in the adjacent lane to which the subject vehicle change the lane from the originating lane over a finite prediction horizon. If the minimum longitudinal clearance between the subject vehicle and the vehicle in the adjacent lane is larger than the minimum safety longitudinal clearance over a finite prediction horizon, then the collision between the subject vehicle and the vehicle in the adjacent lane would be avoided over a finite prediction horizon. Therefore, the lane change of the subject vehicle could be permitted and the desired driving mode could be determined as a lane change mode. On the contrary, if the minimum longitudinal clearance between the subject vehicle and the vehicle in the adjacent lane is smaller than the minimum safety longitudinal clearance over a finite prediction horizon, there could be a collision between the subject vehicle and the vehicle in the adjacent lane during a finite time-horizon and the lane change of the subject vehicle should not be permitted. The decision process of the environmental envelope for a lane change mode is described in Fig. 4-(b).



(a) Lane keeping mode ***(b) Lane Chang mode***

Fig. 4 Decision process of the environmental envelope

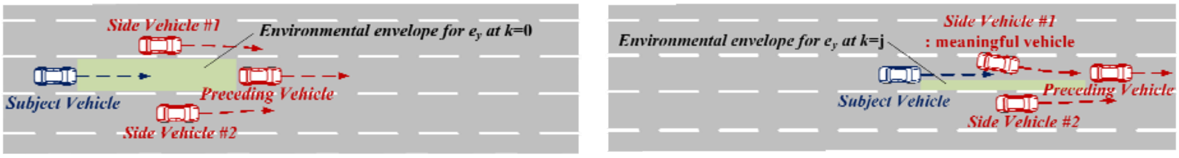
Consequently, the condition of limitation of the lateral deviation, e_y, to satisfying the environmental envelope can be written as follows:

$$\begin{aligned}&H_{env}\cdot x(k)\le G_{env,upper,bound}(k), \quad k=1,\ldots,N_p\\&G_{env,lower,bound}(k)\le H_{env}\cdot x(k), \quad k=1,\ldots,N_p\\&where,\\&\quad H_{env}=\begin{bmatrix}0&0&0&1\end{bmatrix}\end{aligned} \tag{14}$$

Before the determination of the environmental envelope to guarantee the longitudinal safety, we need to define the state of the target vehicle for the control of the longitudinal acceleration. In the case of a lane keeping mode, if the width of the environmental envelope for e_y over a finite prediction horizon is large enough, it means that possible behaviors of surrounding vehicles in the adjacent lane are predicted to keep their lane. Then the preceding vehicle in the originating lane is chosen as the target vehicle for the control of the longitudinal acceleration. If there is no preceding vehicle in the originating lane or the clearance between the subject vehicle and the preceding vehicle is too far, then the virtual vehicle to follow the desired velocity is chosen as the target vehicle for the control of the longitudinal acceleration.

On the other hand, one of adjacent vehicles could be expected to approach to the originating lane of the subject vehicle or change the lane into the originating lane of the subject vehicle. In this case, the width of the environmental envelope for e_y could be smaller than minimum safety width. It means that the subject vehicle could not keep the lane only with the steering maneuver. Generally, when drivers recognize that the neighboring vehicle in the adjacent lane is entering into the lane of the subject vehicle, drivers generally tend to release the throttle pedal or apply the brakes to decelerate [16]. According to the previous research [16], the target vehicle is generated by combining the preceding vehicle in the originating lane and the meaningful vehicle in the adjacent lane. Based on this research, the clearance and relative speed between the subject vehicle and the meaningful vehicle in the adjacent lane are integrated with those between the subject vehicle and the preceding vehicle in the originating lane for the generation of the target vehicle's information. For instance, if the width of the environmental envelope for

e_y at the prediction time step j is expected to be smaller than minimum safety width as shown in Fig. 8, then the weighting factor, ω_{LK}, to determine the target vehicle's state for the longitudinal acceleration control is determined as shown in (15).



(a) Environmental envelope at the prediction time step k = 0

(b) Environmental envelope at the prediction time step k = j

Fig. 5 Determination of the target vehicle for the longitudinal acceleration control in a lane keeping mode

$$\begin{aligned} \omega_{LK} &= f(n, \mathbf{Risk}), \qquad (0 < \omega_{LK} \le 1) \\ &= \max\left(\max\left(\left. \frac{\min\left(TTC^{-1}, TTC^{-1}_{th}\right)}{TTC^{-1}_{th}} \right|_n, 1 - \left. \frac{\min\left(\mathbf{x}, \mathbf{x}_{th}\right)}{\mathbf{x}_{th}} \right|_n \right) \right) \end{aligned} \tag{15}$$

where, TTC means the time to collision and $\mathbf{x}$ indicates the non-dimensional warning index [1]. n in (15) indicates the prediction time step at which the width of the environmental envelope for e_y is smaller than minimum safety width.

Consequently, the integration between the preceding vehicle in the originating lane and meaningful vehicle in the adjacent lane is defined as shown in (16).

$$\begin{bmatrix} C_x(t) \\ v_{x,\mathbf{target}}(t) \\ a_{x,\mathbf{target}}(t) \end{bmatrix} = \omega_{LK} \cdot \begin{bmatrix} C_{x,\text{meaningful}}(j|t) \\ v_{x,\text{meaningful}}(j|t) \\ a_{x,\text{meaningful}}(j|t) \end{bmatrix} + (1-\omega_{LK}) \cdot \begin{bmatrix} C_{x,inlane}(t) \\ v_{x,inlane}(t) \\ a_{x,inlane}(t) \end{bmatrix} \tag{16}$$

In the case of a lane change mode, the target vehicle's states are determined by the integration between the preceding vehicle and the surrounding vehicle in the adjacent lane of the lane change direction. For instance, if the lane change direction is left, then the target vehicle's states are determined by the integration between the preceding vehicle in the originating lane and the surrounding vehicle in the left lane. The weighting factor for the integration in a lane change mode, ω_{LC}, is defined as shown in (17). Then the integration for the determination of the target vehicle's states to control a longitudinal acceleration during a lane change mode is defined as shown in (18).

$$\omega_{LC} = \frac{|e_y|}{2/3 \cdot W_{road}}, \qquad (0 < \omega_{LC} \le 1) \tag{17}$$

$$\begin{bmatrix} C_x(t) \\ v_{x,\mathbf{target}}(t) \\ a_{x,\mathbf{target}}(t) \end{bmatrix} = \omega_{LC} \cdot \begin{bmatrix} C_{x,sidelane,LC} \\ v_{x,sidelane,LC} \\ a_{x,sidelane,LC} \end{bmatrix} + (1-\omega_{LC}) \cdot \begin{bmatrix} C_{x,inlane}(t) \\ v_{x,inlane}(t) \\ a_{x,inlane}(t) \end{bmatrix} \tag{18}$$

where, the subscript 'side-lane' means the vehicle in the adjacent lane to which the subject vehicle changes the lane from the originating lane and W_{road} is the road width which could be known from the vision sensor.

After the determination of the state of the target vehicle for the control of the longitudinal acceleration, then we could define the environmental envelope to guarantee the longitudinal safety. In order to avoid the collision over a finite prediction horizon, the clearance between the subject vehicle and the target vehicle should be larger than minimum safety longitudinal clearance, $C_{x,safe}$, as shown in (19).

$$C_x(k|t) \ge C_{x,safe}, \quad k = 1, \ldots, N_p \tag{19}$$

To satisfy the condition described in (19), the constraint of the distance error between the actual clearance and desired clearance could be defined as follows:

$$\Delta d(k|t) \ge C_{x,safe} - C_{x,des}(k|t) = -\tau_h \cdot v_x, \quad k = 1, \ldots, N_p \tag{20}$$

Moreover, for the improvement of the longitudinal safety, the relative speed between the subject vehicle and the target vehicle should be larger than the threshold of the relative speed, $\Delta v_{x,\min}$, as shown in (21).

$$\Delta v_x(k|t) \geq \Delta v_{x,\min}, \quad k = 1, \ldots, N_p \tag{21}$$

Consequently, the environmental envelope to guarantee the longitudinal safety could be represented as the linear inequality as shown in (22).

$$H_{long} \cdot x_{long,d}(k|t) \geq G_{long,\min}, \quad k = 1, \ldots, N_p \quad where, \; H_{long} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; G_{long,\min} = \begin{bmatrix} -\tau_h \cdot v_x \\ \Delta v_{x,\min} \end{bmatrix} \tag{22}$$

V. Robust MPC based Controller design

As mentioned in Section I, distributed control architecture which is composed of the lateral control law based on robust MPC approach and the longitudinal control law based on robust MPC approach is adopted. In this research, the sampling time, T_s, is chosen as 0.1 second and the length of the prediction horizon, N_p, is chosen as 20. These receding horizon optimization problems are solved at each time step and the first terms of the optimal control sequences are applied to the system. Then receding horizon optimization problems for a shifted prediction horizon are solved to obtain new optimal control inputs at next time step. To solve MPC problem in MATLAB, CVXGEN which is designed to be utilizable in MATLAB is used as solver [17]. The MPC problem is defined using CVXGEN syntax, and the CVXGEN returns convex optimization solver for the defined optimization solver for the defined optimization problem.

A. Background on Robust Model Predictive Control

In this section, we present the background on robust MPC which is used to decide the desired control inputs for the robust control performance. The control problem based on robust MPC is classified into a feedforward control input for the nominal system and a linear feedback control input to reduce the error between the actual state and the nominal state predicted by model of the plant.
Then the control law can be written as follows:

$$u(k) = \bar{u}(k) + K(x(k) - \bar{x}(k)) = \bar{u}(k) + Ke \tag{23}$$

where, $K \in \mathbf{R}^{m \times n}$ is the linear state feedback gain and $e := (x(k) - \bar{x}(k))$ is the error between the actual state and the predicted nominal state. In this paper, the control law of the state feedback gain is LQR.

B. Desired Steering Angle Decision

As mentioned above, in order to obtain the desired steering angle to keep the vehicle in the safe driving envelope while satisfying the robustness of the control performance under model uncertainties and exogenous disturbances, a feedforward steering input for the nominal lateral dynamics model and a feedback steering input for the compensation of the error between the actual states and the predicted nominal states should be integrated. For the determination of a feedforward steering input, we design the cost function as follows:

$$\begin{gathered} J = \sum_{k=1}^{N_p-1} \bar{x}_{lat}(k|t)^T W_{\cos t,lat}\, \bar{x}_{lat}(k|t) + R_{lat,\Delta u} \sum_{k=0}^{N_p-2} \left\| \bar{u}_{lat}(k+1|t) - \bar{u}_{lat}(k|t) \right\|_2 + \left(H\, \bar{x}_{lat}(N_p|t) - y_{des} \right)^T W_{N_p} \left(H\, \bar{x}_{lat}(N_p|t) - y_{des} \right) + R_{lat} \sum_{k=0}^{N_p-1} \left\| \bar{u}_{lat}(k|t) \right\|_2 \\ where, \\ H = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{gathered} \tag{24}$$

where, $W_{\cos t,lat}$ is predefined weighting matrix, which penalize the differences between states and zero, W_{N_p} is predefined weighting matrix to reduce the differences between the final position of the vehicle over a finite prediction horizon and the desired position, R_{lat} and $R_{lat,\Delta u}$ are predefined weighting matrices for the reduction of magnitudes of steering angle control sequences and the rate of change in steering angle control sequences respectively. These matrices are positive-definite symmetric. W_{N_p} is defined as shown in (25).

$$\begin{aligned} y_{des} &= \begin{bmatrix} 0 & W_{road} \end{bmatrix}^T \; : \; Left\;\; Lane\;\; Change \\ y_{des} &= \begin{bmatrix} 0 & -W_{road} \end{bmatrix}^T \; : \; Right\;\; Lane\;\; Change \end{aligned} \tag{25}$$

Since the actuator has a limitation to operate, the control input and there derivatives need to be constrained. These constraints are given as follows:

$$\begin{aligned}&\left|u_{lat}(k|t)\right| \le u_{lat,\max}, && k=0\ldots N_p-1\\ &\left\|u_{lat}(k+1|t)-u_{lat}(k|t)\right\|_{\infty} \le S_{lat}, && k=0\ldots N_p-2\end{aligned} \tag{26}$$

where, $u_{lat,\max}$ is the maximum magnitude of the steering control input and S_{lat} is the maximum magnitude of the rate of change of the steering control input.

In order to ensure the stability of the vehicle, the side slip angle and lateral acceleration should be restricted for the stability of the vehicle. Therefore the condition for the stability of the vehicle can be written as follows:

$$\left|\beta(k|t)\right| \le \beta_{\max} = \tan^{-1}\left(0.02\mu g\right), \quad k=1,\ldots,N_p \tag{27}$$

$$\left|\gamma(k|t)\right| \le \gamma_{\max} = \frac{A_{y,\max}}{v_x}, \quad k=1,\ldots,N_p \tag{28}$$

where, μ denotes tire-road friction coefficient and $A_{y,\max}$ is the threshold of the lateral acceleration, which is chosen as $8\mathrm{m/s}^2$.

The constraints for the stability of the vehicle which are defined in (27) and (28) can be represented as the linear inequality as shown in (29).

$$\left|H_{veh}\cdot x_{lat}(k|t)\right| \le G_{veh,\max}, \quad k=1,\ldots,N_p \quad where,\ H_{veh}=\begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix},\ G_{veh,\max}=\begin{bmatrix}\beta_{\max}\\ \gamma_{\max}\end{bmatrix} \tag{29}$$

Then MPC problem for the determination of the feedforward steering input could be defined by combining (5), (14), (24), (26) and (29) as follows:

$$\begin{aligned}&\min\ (24)\\ &s.t.\ (5),(14),(26),(29)\end{aligned} \tag{30}$$

In order to design the robust MPC while reducing complexity, the effect of model parameter uncertainties and exogenous disturbances on the linear dynamics model in (5) is represented as an additive equivalent disturbance. Then the lateral dynamics model including the additional disturbance term is written as follows:

$$x_{lat}(k+1) = A_{lat,d}x(k) + B_{lat,d}u(k) + F_{\rho,lat,d}\rho_{ref}(k) + w_{lat,eq} \tag{31}$$

where, $w_{lat,eq} \in \mathbf{R}^{4\times 1}$ is the additive equivalent disturbance on the lateral dynamics model. The equivalent disturbance $w_{lat,eq}$ is unknown but assumed to be bounded as shown in (32).

$$w_{lat,eq} \in W_{lat},\ \left|w_{lat,eq}\right| \le \left[0.05, 0.05, 0.5\cdot\frac{\pi}{180}, 0.1\right]^T \tag{32}$$

C. Desired Longitudinal Acceleration Decision

Similar to the lateral control law, the longitudinal control law should be designed to obtain the desired longitudinal acceleration to keep the vehicle in the safe driving envelope while ensuring the robust control performance. Therefore the desired longitudinal acceleration is determined by combining a feedforward input for the nominal longitudinal dynamics model and a feedback input to attenuate the effect on the system by model parameter uncertainties or external disturbances.

In order to determine the feedforward control input for the longitudinal control of the vehicle, we design the cost function as shown in (33).

$$J_{long} = \sum_{k=1}^{N_p}\bar{x}_{long}(k|t)^T W_{\cos t,long}\,\bar{x}_{long}(k|t) + R_{long}\sum_{k=0}^{N_p-1}\left\|\bar{u}_{long}(k|t)\right\|_2 + \left|\bar{u}_{long}(0|t)-a_x\right| + R_{long,\Delta u}\sum_{k=0}^{N_p-2}\left\|\bar{u}_{long}(k+1|t)-\bar{u}_{long}(k|t)\right\|_2 \tag{33}$$

where, $W_{\cos t,long}$ is predefined weighting matrix for the minimization of the differences between states and zero, R_{long} is predefined weighting matrix to reduce the magnitudes of longitudinal acceleration sequences and $R_{long,\Delta u}$ is predefined weighting matrix to prevent abrupt change of longitudinal acceleration in sequences. These weighting matrices are positive-definite symmetric.

The constraints on the range of the longitudinal acceleration control input and change rate during a finite prediction horizon are written as follows:

$$\begin{aligned}&\bar{u}_{long,\min} \le \bar{u}_{long}(k|t) \le \bar{u}_{long,\max}, && k=0\ldots N_p-1\\ &\left\|\bar{u}_{long}(k+1)-\bar{u}_{long}(k)\right\|_{\infty} \le \bar{S}_{long}, && k=0\ldots N_p-2\end{aligned} \tag{34}$$

where, $\bar{u}_{long,\min}$ and $\bar{u}_{long,\max}$ are the minimum and maximum magnitude of the longitudinal acceleration control input respectively. $\overline{S}_{long}$ is the maximum magnitude of the rate of change of the longitudinal acceleration control input.

Then MPC problem for the determination of the feedforward longitudinal acceleration input could be formulated by combining (10), (22), (33) and (34) as follows:

$$\begin{aligned} &\min \ (33) \\ &s.t. \ (10), (22), (34) \end{aligned} \tag{35}$$

In order to determine a feedback control input for the longitudinal control of the vehicle, an additive equivalent disturbance is included in (10) to represent the effect on the system by model parameter uncertainties or external disturbances.

$$x_{long}(k+1) = A_{long,d}x_{long}(k) + B_{long,d}u_{long}(k) + F_{long,d}a_{x,\text{target}}(k) + w_{long,eq} \tag{36}$$

where, $w_{long,eq} \in \mathbf{R}^{3\times1}$ is the additive equivalent disturbance on the longitudinal dynamics model. Similar to the equivalent disturbance on the lateral dynamics model, it is assumed that the equivalent disturbance on the longitudinal dynamics model, $w_{long,eq}$, is unknown but bounded as shown in (37).

$$w_{long,eq} \in W_{long}, \ \left|w_{long,eq}\right| \le \left[0.05, 0.1, 0.05\right]^T \tag{37}$$

VI. Vehicle test results

The proposed automated driving control algorithm is evaluated through computer vehicle tests. In order to evaluate the proposed algorithm on a real test vehicle, Hyundai-Kia Motors K7 is used as a test vehicle platform. Figure 6 shows the test vehicle configuration. In order to measure DLC, heading angle and road curvature, a Mobileye camera system is equipped on the test vehicle. The proposed algorithm has been implemented on "dSPACE Autobox", which is used for the real-time application and equipped with a DS1005 processor board. Delphi radars are equipped on the test vehicle to perceive surrounding environments. The hardware components mentioned above communicate through a CAN bus.

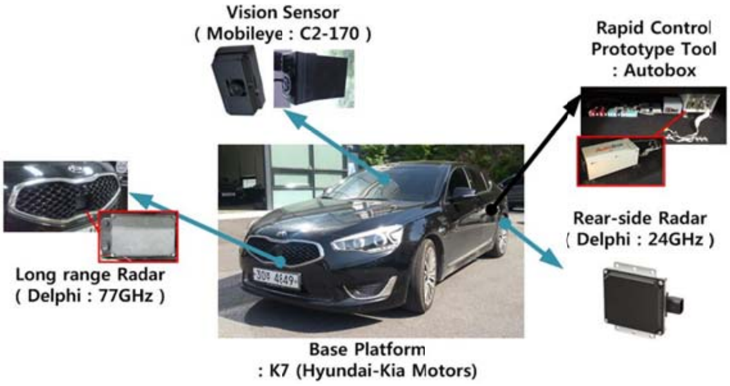


Fig. 6 Test vehicle configuration

The test track is a straight road. The road-tire friction coefficient is assumed to be 0.85, since the road of the test track is a dry asphalt road. Two cases of experiments have been conducted. In order to evaluate the performance of the proposed algorithm under lane change situation, the scenario of experiment is designed to evaluate the performance of the proposed algorithm under overtaking situation as shown in Figure 7.

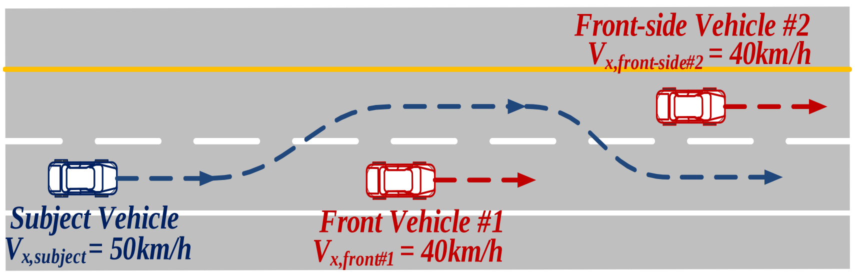
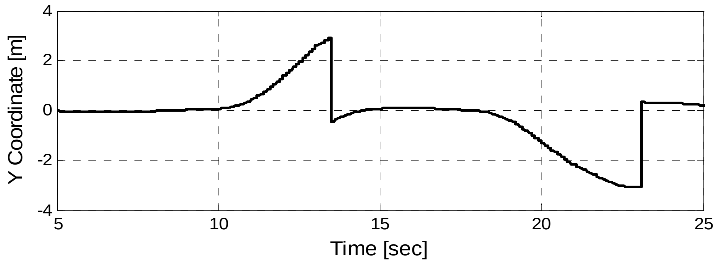
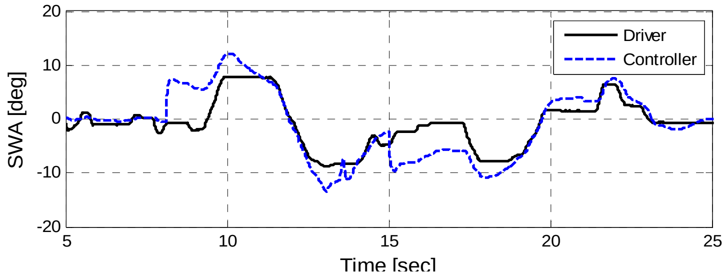


Fig. 7 Experiment scenario for overtaking

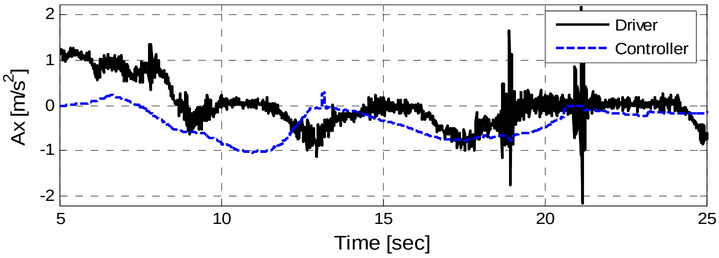
The simulation results are presented in Figure 8. As shown in Figure 8, it can be known that the controller shows quite similar performances to the human driver while changing lane. Based on these results, it has been shown that the proposed algorithm could reflect human driver's driving characteristics. It means that the proposed algorithm could provide acceptable ride comfort in general driving situations. Since lateral offset is measured by camera sensors, lateral offset is plotted as discontinuous as shown in Figure 8-(a). Figure 8-(b) and (c) depict steering angle and longitudinal acceleration comparing results between the human driver and the controller. Lateral acceleration has reasonable magnitude as shown in Figure 8-(d).



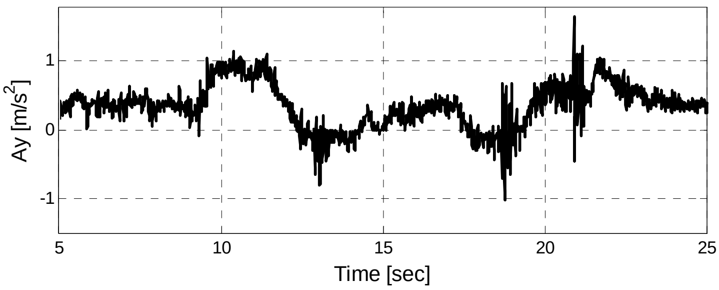
(a) Lateral offset



(b) Comparison of the steering angle between the driver and the controller



(c) Comparison of the longitudinal acceleration between the driver and the controller



(d) Lateral acceleration

Fig. 8 Comparison between the driver and the proposed algorithm under overtaking situation

VII. Conclusion

A robust MPC based vehicle speed and steering control algorithm has been developed to enhance safety and ensure constraint satisfaction under model uncertainties and external disturbances. In order to cope with potential risky situation, not only current states of surrounding environment but also potential risky behaviors of that during a finite time horizon are considered simultaneously in determining the desired driving mode and the safe driving envelope. Then distributed control architecture based on robust MPC approach is used to determine the desired steering angle and desired longitudinal acceleration separately while satisfying reliability and reducing a computational burden.

In order to verify the effectiveness of the proposed control algorithm, computer simulations have been conducted. The simulation results show that the proposed control algorithm enhances safety with respect to the potential risk and provides permissible ride comfort. Furthermore it has been shown that robust vehicle control performance can be obtained in the presence of additional disturbances by using the proposed algorithm.

In the future, we should verify the performance of the proposed algorithm via vehicle tests.

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