HERTZ CONTACT MODEL TO ESTIMATE PEDIATRIC HEAD IMPACT RESPONSE VARIABLES

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ABSTRACT

This paper describes a study to develop and validate methodology for simulating human infant head impact using the Hertz contact model. The study had two objectives. The first was to simulate Aprica 2.4 dummy head -rigid plate impact using the Hertz contact model to estimate head response variables. Model estimates were then compared with corresponding test variables. The second objective based on success of the first, was to evaluate the feasibility of using Hertz contact model to simulate human pediatric head – rigid plate impact at contact velocities ranging from 1.7 m/s to 6.26 m/s.

During objective 1 of this study, known geometric and material properties of the Aprica dummy head and steel plate were used as the Hertz model parameters. Model estimates of peak acceleration, peak head compression, pulse width, and time to peak displacement were compared with corresponding test data for contact velocity of 2.3 m/s. Percentage differences in response variables were: peak acceleration – 2.5*;* peak head compression – 1.3; pulse width – 6.1; and time to peak compression - 2.2.

During objective 2 of this study, human head impacts were divided into 4 age groups – neonate (under 1-month), 5-months, 9-months and 11-months. Objective 2 was divided into 2 stages – Model building and Model validation. In the Model Building stage, a method was developed to estimate Hertz contact model parameters using human 30 cm drop test data. In the Model Validation stage, the model was used to estimate head response variables for 15 cm, 30 cm and 2 m drops for all four age groups and compared with human test data. Model estimates for peak head acceleration of neonates in 15 cm and 30 cm drop tests differed from average test peak head acceleration by 11%, and 13% respectively. Neonate estimated pulse widths for the same drop heights differed from test average by 0% and 1%. Maximum and minimum differences for 5-, 9-, and 11-month infant model estimates from average test values in 15 cm and 30 cm drops were: 13.47% and 0.44% for peak acceleration and 6.68% and 0.03% for pulse width. Simulation results of 2 m drops of 5-month, 9-month, and 11-month old heads indicated that estimated head Jerk (rate of change of acceleration) was very close to human test results. Since the pediatric heads sustained fractures, it was not possible to compare peak accelerations.

The model reproduced, very closely, the static force-deformation curve for 5-month old but provided poor estimates for some neonates. Model reproduced finite element model results for 30 cm drop test for 5-month old head on to concrete and hard foam. The proposed model and methodology provide a simple procedure to estimate pediatric head acceleration, head deformation and pulse width for contact velocities ranging from quasi-static to 6.3 m/s onto rigid and soft surfaces.

INTRODUCTION

Availability of head impact models greatly facilitate understanding of head injury causation, determination of injury tolerance values, and design of dummy heads and head protective gear. Typically, models are based on results of several well-designed cadaver head impact tests. However, ethical and societal concerns prevent researchers from conducting requisite number of tests with pediatric heads to evaluate their material properties and to estimate their impact response. Limited isolated pediatric cadaver head testing has been conducted by Prange (2003) and Loyd (2011). They dropped isolated pediatric cadaver heads onto a rigid plate. Weber, et al. (1984, 1985) conducted full body child cadaver drop tests. They dropped un-instrumented cadavers onto rigid, and padded surfaces. However, these tests do not cover the full age and size range of the pediatric population.

Researchers have attempted to reconstruct free falls attempting to fill the information gap in pediatric head impact response. Snyder et al (1963, 1977) documented falls from heights up to 11m in their attempt to estimate the relationship between injury severity, fall height, and type of contact surface. They used a combination of detailed

medical documentation, scene investigation, and lumped mass modeling to relate fall heights and injury for free falls on to surfaces of varying stiffness.

Other researchers, Li, et al, (2015), Li, et al, (2013), Coats (2007), Ibrahim (2012), Roth (2008 and 2010) among others, have developed detailed finite element models of infant and child skull and brain. Some of these models have been validated against human impact data. Brooks, et al (2018) have provided a review of pediatric finite element models.

Engineers have idealized impact tests such as head drop tests as Sphere – Half-space impact using the Hertz contact model. This kind of modeling is used in a variety of fields such as evaluation of delamination propensity of composite wings due to small runway objects hitting the wings (Olsson, 2003), impact of biological cells (Biersack, 2010), flow of pharmaceutical powders (Antypov and Elliott, 2011). Abrate (2001) provides an overview of this class of problems and solution methods.

This study attempts to further fill the gaps in pediatric head impact data by evaluating use of the Hertz contact model to simulate human infant head – rigid surface impact. A successful attempt would provide designers with an easy to use tool to explore effects of impact in the pediatric population not covered by cadaver tests.

METHODS

The following topics will be discussed in this Section:

1. Assumptions used in this study.

2. Methodology for simulation of Aprica dummy head – rigid plate impact using the Hertz contact model.

    a. Simulation model development methodology

    b. Parameter sensitivity study methodology.

    c. Methodology used to reduce dummy drop test data.

3. Simulation of pediatric head – rigid plate impact.

    a. Model Building: This sub-section will provide details of methodology used to estimate Young's modulus of pediatric heads. The Hertz model parameters are; radii, Young's moduli, and Poisson's ratios of the two contacting partners and contact velocity. The value of material and geometric variables for the rigid (Aluminum) plate are well known. Pediatric head radius was calculated from its circumference. Head Poisson's ratio was estimated from literature sources. However, pediatric heads are composed of many deformable features such as cartilaginous bones, sutures, fontanels, etc. So, there was no obvious method to calculate the Young's modulus of this structure under impact. Young's modulus was therefore estimated using infant experimental data in this stage.

    b. Model Validation: Methodology used to validate the model developed above will be discussed in this sub-section.

1. Study assumptions

The following assumptions were made in this study:

- The head was modeled as a sphere whose radius was calculated from individual head circumferences listed in Loyd (2011).

- The plate was flat and rigid, and only the head deformed during impact.

- Air resistance was neglected in calculating contact velocity from drop heights.

2. Simulation of Aprica dummy head – rigid plate impact

The design of the Aprica 2.4 dummy has been described in Rangarajan, et al (2002), and head drop test methodology and results have been described in Rangarajan, et al. (2017). Methodology adopted to simulate dummy head drop tests will be described below.

2.a. Simulation model development

Hertz contact model equation governing impact of a sphere against a large, rigid plate where all the deformation is restricted to the area of contact, can be written as:

$$m\ddot{x} = -kx^{3/2} \qquad \text{(Equation 1)}$$

With initial conditions, x(0) = 0 and $\dot{x}(0) = \sqrt{2gh}$, the velocity at initial contact.

The term "k" which is the Contact Stiffness is defined as:

$$k = \frac{4}{3}E_{contact}\, R_{contact}^{1/2} \qquad \text{(Equation 2)}$$

And Contact modulus (E$_{contact}$) and Contact Radius (R$_{contact}$) are defined as:

$$\frac{1}{E_{contact}} = \frac{1-v_{plate}^2}{E_{plate}} + \frac{1-v_{sphere}^2}{E_{Sphere}} \qquad \text{(Equation 2a)}$$

$$\frac{1}{R_{contact}} = \frac{1}{R_{plate}} + \frac{1}{R_{Sphere}} \qquad \text{(Equation 2b)}$$

Where v = Poisson's ratio.

Equation (1) was solved for the Aprica 2.4 dummy head drop test. Dummy head mass (0.85 kg) and circumference (0.34 m) were obtained from Rangarajan, et al. (2002). Young's modulus for the Shore 50A head was calculated using:

log E = (0.0235*(S))-0.6493, where S = Shore A value (50)   (Equation 3)

Young's modulus was calculated to be 3.42 MPa. Contact velocity was computed from known drop height, neglecting air friction using the formula-

$$v = \sqrt{2gh}. \qquad \text{(Equation 4)}$$

Euler's method was used to solve equation (1) with a time step of 0.000001s. Solution was implemented in a spreadsheet. Time step was varied by an order of magnitude to confirm convergence. Time-wise plots of estimated head acceleration were output by the model. Acceleration was integrated to estimate variation of velocity and displacement with time. Since the plate was rigid, it was assumed that displacement of the head after contact represented its compression. Table 1 lists base simulation model parameters.

Table 1
Model Parameters for Base Simulation

Parameter	Plate	Head
Young's modulus Steel	210 GPa,	3.42 MPa
Radius	∞	0.0541m
Poisson's Ratio	0.2	0.48

2.b. Parameter sensitivity study

Hertz's model requires contact velocity, radius, and material properties of the plate and striking object. Of these, contact velocity is calculable with a degree of certainty. Material properties of the plate and its radius are known to a high degree of certainty. Dummy head was molded from 2-part Urethane as a prototype. Under these circumstances, the Shore hardness can vary + / - 4 around the base value of Shore A 50. This variation would cause approximately 25% change in modulus. I assumed that head circumference measurement by hand calipers could also vary by a maximum of 25%. Therefore, a parametric sensitivity was conducted first holding all other parameters constant and varying E_{head} by +/- 25% over its base value. R_{head} was also varied by +/- 25% while holding all other parameters at base value. Results of these sensitivity studies are shown in RESULTS section.

2.c. Reduction of dummy head drop test data

A tri-axial accelerometer cluster was used to measure dummy head acceleration at 10 kHz and filtered according SAE J-211. Head accelerations are shown in Rangarajan, et al (2017). Mean head acceleration from four isolated head drop tests was calculated. Mean acceleration was integrated to estimate head displacement and velocity. Time-wise variation of test head acceleration, velocity and displacement were compared with corresponding model estimated variables.

3. Simulation of pediatric head – rigid plate impact.

Simulation was divided into 2 stages as discussed below.

3.a. Model building – Development of contact model for infant – rigid plate impact

As discussed previously, it was necessary to estimate modulus of the infant head, a parameter required in the Hertz model (see Equation 2a). Pediatric drop test data from Loyd (2011) was used to build the model. The process was:

- Obtain average pulse width and average peak acceleration for 30 cm drop tests for four age groups - neonates (<1m old), 5-, 9- and 11- month old pediatric heads. For each age group, pulse width and peak acceleration in all drop directions (vertex, occipital, forehead, left parietal, right parietal) were included in the calculation of average.

- Use calculated average pulse width, average head mass, average head circumference of each pediatric age group, and, known pulse width, head circumference, head mass, and modulus of the Aprica dummy head to calculate Contact Stiffness as shown in the Appendix. The calculation is based on the formula for Pulse width in Hertz contact model which is (Abrate, 2001):

$$\tau = 3.2145 \left(\frac{m^2}{Vk^2}\right)^{1/5} \qquad \text{(Equation 4)}$$

The rigid surface in these simulations was modelled as an Aluminum plate with modulus of 73.6 GPa and Poisson's ratio of 0.35. This completed Model Building.

3.b. Model Validation – Compare model estimates with infant head drop and static loading test data

To my knowledge, this is the first time the Hertz contact model has been validated against human pediatric test data. Therefore, I have attempted to validate the model extensively. Validation runs were made to evaluate accuracy of estimates across a range of infant ages and contact velocities. Simulation were run to evaluate accuracy of model estimates in quasi-static tests. The accuracy of model estimates was also evaluated for contacts against rigid Aluminum and steel plates, concrete, and against softer foam surface. Pediatric model estimates were compared with neonate infant head drop test and static compression data from Loyd (2011) to validate the model.

Results of these simulations will be discussed in the next section.

RESULTS

Results of the following validation tests will be discussed in this Section.

1. Model of the Aprica dummy head drop test was exercised and model estimates were compared to dummy test head response variables (Table 2).

    a) Base simulation results (Table 2)

    b) Parameter sensitivity simulation results (Tables 3 and 4)

2. Results of pediatric model building.

3. Validation of the pediatric Hertz contact model:

    a) Exercise neonate model developed in the Model building phase to simulate 15- and 30- cm drops and compare head response variables with neonate test data (Tables 5 and 6).

    b) Exercise 5-, 9- and 11- month child head model for 15- and 30- cm (Tables 7 and 8) and compare with test data.

    c) Compare model estimates of 2 m drops for 5-, 9- and 11-month old subjects with corresponding test data (Table 9, and Figures 4, 5 and 6)

    d) Compare estimated contact force, head deformation and time to peak deformation for neonate P03 with head drop test results (Table 10).

    e) Use estimated Contact Stiffness to calculate contact force for defined head deformation in quasi-static tests. Compare the resulting force – deflection data with corresponding human pediatric data (Figures 7 and 8).

    f) Model of 5-month old head was exercised to simulate various height drops onto hard foam and concrete Model estimates were compared with finite element model estimates (Li, et al. 2013). This comparison is presented in Figure 9, and Table 11.

4. Effects of idealizing the pediatric head as a sphere on estimated head accelerations is discussed (Table 12, 13, 14 and 15).

1. Comparison of test and model results for Aprica dummy isolated head impact

Model estimates (acceleration, head compression and velocity) from isolated head drop tests were compared with corresponding Aprica dummy isolated head drop tests. This was done to establish the applicability of Hertz contact model for non-elastic sphere in a Sphere – Half-space impact. The sphere's material and geometric properties were in the range of human pediatric subjects making this comparison meaningful to the objectives of this paper. Results of these simulations will be discussed first followed by simulation results of human head drop tests.

<u>**1.a. Base Aprica dummy head simulation results**</u>

Isolated head of the Aprica 2.4 dummy was dropped from a height of 0.376 m onto a steel plate such that the forehead contacted the plate. Drop test methodology and data collection techniques were discussed in Rangarajan, et al. (2017). Isolated dummy head drop tests were similar to infant drop tests. Dummy drops were simple tests where material and geometric properties of the contacting partners were known with a high level of certainty and so these could be used as control tests to investigate applicability of Hertz model to pediatric head impacts. Results of these tests are discussed in this Section.
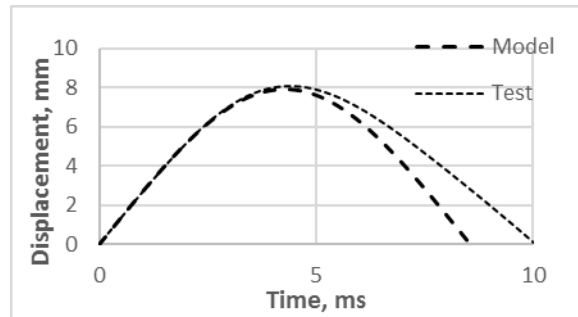
Comparison between test and model estimated acceleration, velocity and compression are shown in Figs. 1, 2 and 3 respectively.
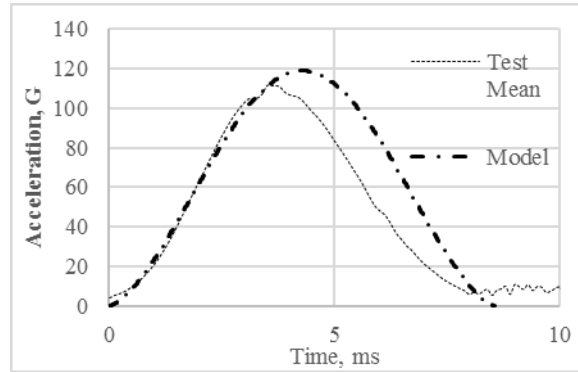
Figure 1: Comparison of test mean head resultant acceleration and model acceleration
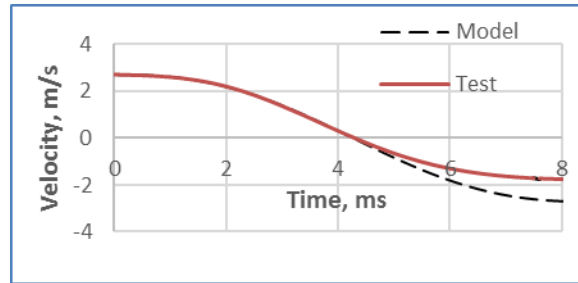
Figure 2: Comparison of test and estimated head velocity

Figure 3: Comparison of test and model head compression

It is seen in Fig. 1 that test and model acceleration are coincident for the majority of deformation phase when the head velocity is being brought to zero from the initial contact velocity. The model acceleration is symmetric about the time velocity reaches zero as Hertz model assumes an elastic collision.

Figure 2 shows that model rebound velocity is equal to contact velocity because Hertzian contact is assumed to be purely elastic. However, the rebound velocity of the elastomeric head is lower than contact velocity leading to a coefficient of restitution of approximately 0.75. This shows that the Hertz contact model provides a good estimate of peak acceleration, peak deformation and pulse width for non-elastic impacts also during the crush phase.

Figure 3 shows that model head deformation follows test data until a maximum is reached. The model head deformation curve is symmetric about the time of maximum deformation whereas the test Aprica head regains it shape later.

Test and model estimated response variables for base simulation are summarized in Table 2.

Table 2
Comparison of test and model estimated head response variable values

Variable	Test Value	Model estimate	Difference [%]
Pulse Width [ms]	8.1	8.6	6.1
Hd Pk Accel [g]	115	119	2.5
Max Comp	8.0	7.9	1.3

1.b. Parameter sensitivity simulation results

Results of parameter sensitivity study are shown in Tables 3 and 4. Column 3 of Table 3 shows that if modulus of the head is increased by 25% over the base value, pulse width is lowered 9%, head peak acceleration is increased by 9% and peak head compression is increased by 8.5%, all increases over their respective base values. Column 4 shows results when head modulus is decreased by 25% over its base value.

Table 3
Variation in simulation estimates with ± 25% change in E_{head}

Variable	Base Value, E_{head}	$E + 0.25 \, E_{head}$	$E - 0.25 \, E_{head}$
Pulse Width [ms]	8.6	7.82 (9%)	9.6 (12%)
Head Pk Accel [g]	119	130 (9%)	105 (12%)
Max Compression [mm]	7.9	7.23 (8.5%)	8.9 (13%)

Column 3 of Table 4 shows that a 25% increase in R_{head} would cause a 5% reduction in pulse width, 4% increase in head peak acceleration and 10% decrease in peak compression. Column 4 provides figures when R_{head} is decreased by 25% over the base value.

Table 4
Variation in simulation estimates with ± 25% change in R_{head}

Variable	Base Value, R_{head}	$E + 0.25 \, R_{head}$	$E - 0.25 \, R_{head}$
Pulse Width [ms]	8.6	8.2 (5%)	9.1 (6%)
Head Pk Accel [g]	119	124 (4%)	112 (6%)
Max Compression [mm]	7.9	7.6 (4%)	8.4 (6%)

Reasonable estimates for head response variables for the Aprica dummy non-elastic head provided the momentum to apply Hertz contact model to pediatric head drops. Results of pediatric model simulation are provided in the next few sections.

2. Results of pediatric head model building

Model building procedure is described in the Appendix. Average head mass, and circumference of the 6 neonates (P03M, P05F, P06M, P07M, P08M and P13F in Loyd, 2011) were 0.534 kg and 0.323 m respectively. Average pulse width for 30 cm drop tests for these neonates was 16.38 ms. Aprica dummy head mass and circumference were 0.85kg and 0.34 m respectively and pulse width was 8.6 ms. Young's modulus and Poisson's ratio of Steel were 2.1GPa and 0.2 respectively and those for the elastomer which was used to mold the head were 3.42 MPa and 0.48 respectively.

The Appendix shows how these values were used to Contact Stiffness (k) and Young's modulus for neonates which were estimated to be $0.129*10^6$ N / $m^{3/2}$ and 0.464 MPa respectively. Contact Stiffness (k) for 5-month, 9-month

and 11-month child respectively were $0.624*10^6$ N / m$^{3/2}$, $0.949*10^6$ N / m$^{3/2}$, and $2.15*10^6$ N / m$^{3/2}$. Young's modulus for these infant heads were estimated to be 1.43 MPa, 2.01 MPa and 4.67 MPa respectively.

3. Validation of Hertz model for pediatric subjects

This Section will present comparison between estimated head response variables and corresponding test data for 15 cm, 30 cm for all age groups, and 2 m drop tests for 5-, 9- and 11-month old heads. In addition, quasi-static plate loading Force – Deflection test data will be compared with corresponding estimates.

3.a. Comparison of estimated and test variables for neonates

Table 5 and 6 present a comparison between test and model estimates of response variables for 6 neonates in the 15 cm and 30 cm drop test respectively. Value of Young's modulus [0.464 MPa] for the average infant was used in these simulations. The model estimates for peak acceleration and pulse width correlated well with average test acceleration and average test pulse width with estimate errors of 11% for peak acceleration and 0% for pulse width. Data shown in Table 6 indicate that errors of approximately the same magnitude are seen in 30 cm drop test estimates. Neonate average head acceleration and pulse width estimates in 30 cm drop tests differ from test values by 13% and 1% respectively.

Table 5
Comparison of estimated neonate response variables for 15 cm drop test with test variable

Subject	Neonate Average	P03M	P05F	P06M	P07M	P08M	P13F
Age, m		0.1	0.03	0.36	-1.58	-0.56	-1.35
Avg Test Accel, G	39.29 ± 9.62	49.8	33.64	33.16	43.7	28.72	46.72
Estimate Accel, G	35.13	38.38	33.76	33.03	38.11	32.66	37.06
Difference in Accel, %	10.59	22.93	-0.36	0.39	12.79	-13.72	20.68
Avg Test pulse width, ms	18.2 ± 5.04	13.06	23.04	22.34	15.42	23.12	12.1
Estimate pulse width, ms	18.29	16.74	19.03	19.45	16.86	17.68	17.34
Difference in pulse width, %	0	-28.18	17.4	12.54	-9.34	23.53	-43.31

Table 6
Comparison of estimated neonate response variables for 30 cm drop test with test variable

Subject	Neonate Average	P03M	P05F	P06M	P07M	P08M	P13F
Age, m		0.1	0.03	0.36	-1.58	-0.56	-1.35
Avg Test Accel, G	61.52 ± 16.79	81.22	51.76	52.94	74.94	47.96	60.3
Estimate Accel, G	53.25	58.17	51.17	54.59	62.88	53.87	61.14
Difference in Accel, %	13.44	28.28	1.14	-3.12	16.09	12.32	-1.30
Avg Test pulse width, ms	16.38 ± 3.88	12.28	19.78	19.28	12.94	20.5	13.5
Estimate pulse width, ms	17.07	15.62	17.76	18.15	16.73	18.36	16.18
Difference in pulse width, %	0.74	10.21	5.86	-29.29	10.44	-19.85	0.74

However, when peak acceleration and pulse width are estimated for individual infants, keeping the Young's modulus constant at value derived previously, i.e., 0.464 MPa, maximum error in estimates is 23% for acceleration of P03M, and 43% for pulse width for P13F in 15 cm drop simulations as seen in Table 5. The same trend is seen in Table 6 which indicates that peak acceleration for neonate P03M differs from test value by 29%. Estimated pulse width for neonate P06M differs from test value by 29%.

3.b. Comparison of estimated and test variables for 5-, 9- and 11-month old heads

Tables 7 and 8 show results of 15 cm and 30 cm drop simulations respectively for the 6-, 9- and 11-month old infants. Errors in estimates for both acceleration and pulse width trend lower for these infants compared to neonates which might be the result of a single head at each age being tested.

Table 7
Comparison of child drop test variables with simulation results for 15 cm drops

Subject	P12M	P14M	P15F
Age, m	6	9	11
Avg Test Accel, G	41.84 ± 3.15	35.48 ± 2.42	60.78 ± 14.93
Estimate Accel, G	46.62	40.26	62.8
Difference in Accel, %	-11.42	-13.47	-3.32
Avg Test pulse width, ms	12.96 ± 1.92	14.96 ± 0.2	10.33 ± 2.37
Estimate pulse width, ms	13.78	15.96	10.23
Difference in pulse width, %	-6.33	-6.68	1.37

Table 8
Comparison of child drop test variables with simulation results for 30 cm drops

Subject	P12M	P14M	P15F
Age, m	6	9	11
Avg Test Accel, G	70.98 ± 8.48	63.08 ± 9.69	87.76 ± 16.73
Estimate Accel, G	70.67	61.03	95.18
Difference in Accel, %	0.44	3.25	-8.45
Avg Test pulse width, ms	12.66 ± 0.99	13.64 ± 0.69	9.22 ± 1.84
Estimate pulse width, ms	12.86	14.89	9.55
Difference in pulse width, %	-0.03	-9.16	-3.58

3.c. Comparison of estimated and test accelerations in 2m drop tests
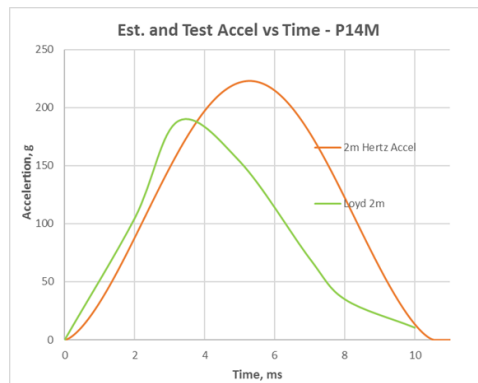
Test and estimated head acceleration for P12M, P14m and P15F heads are summarized in Table 9 and plotted in Figs. 4, 5, and 6 respectively. Figures 4 and 5 indicate that the model estimate for Jerk is very similar to test values in the loading or crush part of the acceleration pulse. Since all three heads fractured during 2m drop tests, it is not possible to compare peak accelerations. In Figures 4, 5 and 6, I have tried to retain salient points of the response plots from Loyd (2011) but test responses plots are not likely to be true through their extant.

Table 9
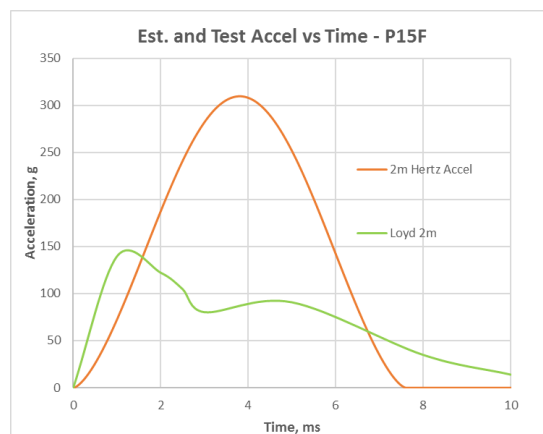**Comparison of test and estimated accelerations in 2m drop tests**

Subject	P12M	P14M	P15F
Age, m	6	9	11
Test Peak acceleration, g	158	155	147
Test Peak force, N	1492	2630	2032
Estimated Peak Acceleration, g	223	207	309
Estimated Peak Force, N	2007	3569	4478
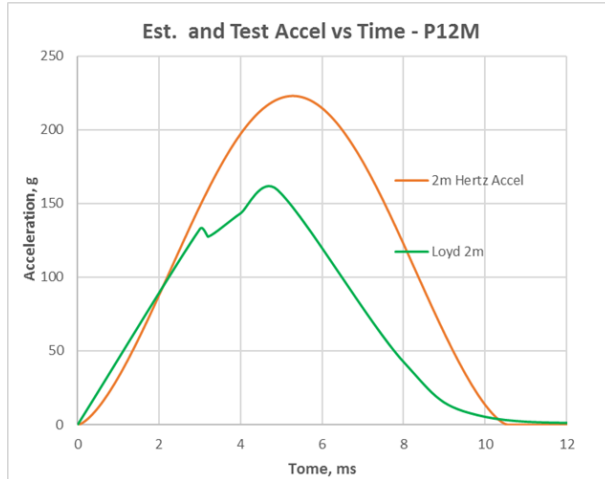
Figure 4: Test and estimated acceleration for P12M in 2m drop – Head fractured

Figure 5: Test and estimated acceleration for P14M in 2m drop test – Head fractured

Figure 6: Test and estimated acceleration for P15F in 2m drop test – Head fractured

Figures 4 and 5 show that estimated acceleration vs time curve very closely follows the test data for P12M and P14M up to the point where head fractures. Correlation for P15F is shown in Figure 6 indicating that estimates are

worse than those for other two heads. Only one test each was conducted with 5-, 9- and 11-month old heads and more information is needed before model estimates can be meaningfully compared with human head responses.

3.d. Comparison of dynamic head compression in neonate tests

Loyd (2011) plotted head deformation and contact forces for 30 cm drop test on P05F head. Table 10 provides a comparison of response variables for this infant head at contact velocity of 2.43 m/s.

Table 10
Comparison of response variables for P05F head in 30 cm drop test

Mode	Peak Accel, g	Peak Compression, mm	Peak contact force, N	Time to Peak accel, ms
Test	51.76	~15	~305	~8
Simulation	51.17	14.7	301	~8.8

3.e. Comparison of response in quasi-static head compression tests

Loyd (2011) conducted quasi-static head compression tests in which the head was held between the platens of MTS machine. Figure 7 shows A-P compression data at 0.3/s loading rate compared with estimated force calculated using Equation (1). Model generated F-D curve is quite similar to test data. Comparison for P03M head was not as good and is shown in Fig. 8. This poor fit between estimated and test data caused errors in estimated acceleration and pulse width in drop tests seen in Tables 5 and 6.
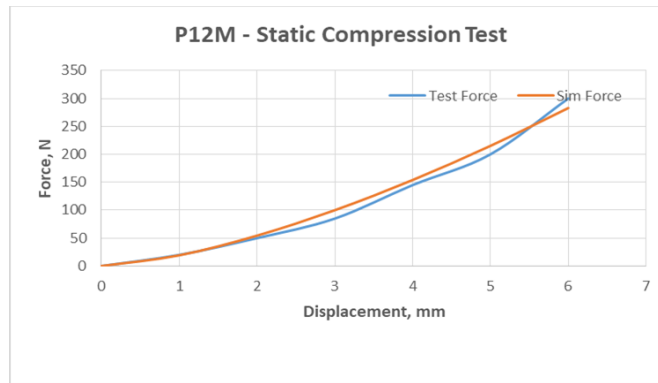
Figure 7: Comparison of quasi-static test and estimated compression for P12M
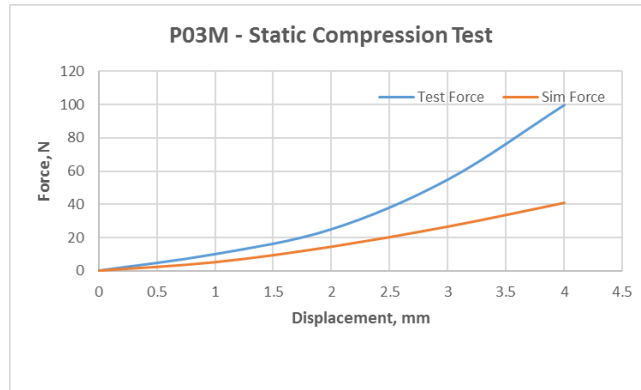
**Figure 8: Comparison of quasi-static test and estimated compression for P03M**

3.f. Comparison of 5-month old FE model and Hertz estimates for impact with soft surfaces

In order to complete the suite of verifications, this study replicated Li, et al. (2013) FE simulations of 6-month old infant head impact against soft foam and concrete. These simulations were conducted to suitability of the pediatric model to simulate head impact against non-rigid surface. Like Li, et al. (2013), Hertz contact model simulations were run for P13M [5-month old]. Results are tabulated in Table 11 and plotted in Fig. 9.

In these simulations, modulus and Poisson's ratio for Concrete were obtained from Li, et al. (2013) and were set in Hertz model as 30 GPa, and 0.15 respectively. Modulus for hard foam was obtained from a plot in the same paper and set at 1.6 MPa in the simulation model. Tabulated data and Fig. 9 indicate that the current model can provide reasonable estimates for peak acceleration when the head contacts a soft surface such as foam whose modulus is approximately 5 orders of magnitude softer than concrete.

Table 11
P13M fall onto concrete and hard foam – comparison of Hertz estimates with FE model

Drop Height, cm	Li Acceleration hard foam	Hertz estimated. hard foam	Li. Acceleration Concrete	Hertz Acceleration concrete
20	~30	41	~50	56
30	~40	53	~75	71
40	~55	63	~85	84
60	!75	81	~120	108
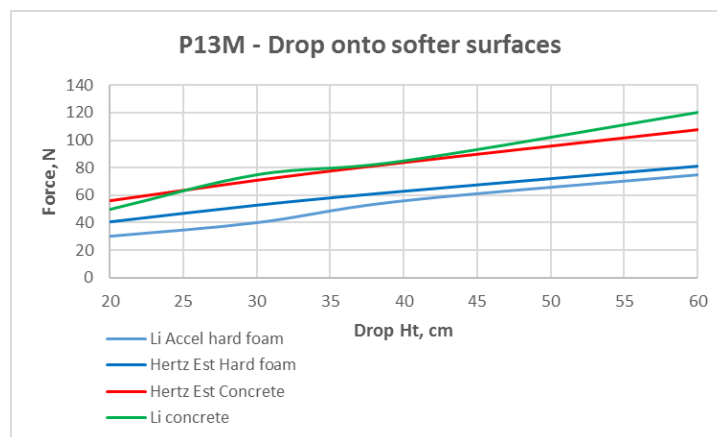
Figure 9: P13M head impact onto to concrete and hard foam

4. Effect of idealizing head as a sphere

Pediatric head has been idealized as a sphere in this study. However, Loyd (2011) has observed that peak head acceleration varies with impact direction. Vertex impacts generally generated the highest acceleration for any head in tests. A sphere will not show the effect of impact direction on head acceleration. Tables 12, 13, 14 and 15 show model estimated acceleration and average acceleration for each neonate impact direction. 5-month old, 9-month old, and 11-month old heads respectively. Neonate head accelerations for each impact direction for each drop height were averaged over 6 subjects.

Table 12 compares estimated accelerations and average test accelerations for each impact direction for neonates. It is seen that estimated accelerations are quite close (approximately 10% difference) to test accelerations for all impact directions except Vertex.

Table 12
Effect of idealizing neonate head as a sphere

Direction	Est. Accel, g	Neonate, avg. accel, g
15 cm Vertex	35.13	46.8 ± 10.5
15 cm Occiput	35.13	37.1 ± 6.76
15 cm Forehead	35.13	38.5 ± 11.5
15 cm Rt. Parietal	35.13	37.1 ± 6.5
15 cm Lt, Parietal	35.13	36.9± 6.8
30 cm Vertex	53.3	75.9 ± 19.9
30 cm Occiput	53.3	57.4 ± 9.7
30 cm Forehead	53.3	59.3 ± 19.4
30 cm Rt. Parietal	53.3	58.2 ± 12.6
30 cm Lt, Parietal	53.3	58.8 ± 11.2

Similar comparison is presented for 5-, 9- and 11-month heads in Tables 13, 14, and 15. It is not valid to draw meaningful conclusions from these data as only one subject has been tested in each age group. However, it is interesting that peak accelerations of P14M, a 9-month head in all impact directions are very close to those of neonates for both 15 and 30 cm drop tests even though both the modulus and CS of this head is nearly double that of the average neonate. This is because the increased head mass modulates the accelerations which is substantiated by model estimates which too are close to neonate estimates.

Table 13
Effect of idealizing 5-month head as a sphere

Direction	Est. Accel, g	Neonate, avg. accel, g
15 cm Vertex	46.6	44.9
15 cm Occiput	46.6	43.7
15 cm Forehead	46.6	44.5
15 cm Rt. Parietal	46.6	37.3
15 cm Lt, Parietal	46.6	38.8
30 cm Vertex	70.7	71.6
30 cm Occiput	70.7	81.0
30 cm Forehead	70.7	57.6
30 cm Rt. Parietal	70.7	78.5
30 cm Lt, Parietal	70.7	66.2

Table 14
**Effect of idealizing 9-month head as a sphere**

Direction	Est. Accel, g	Neonate, avg. accel, g
15 cm Vertex	40.3	35.4
15 cm Occiput	40.3	37.6
15 cm Forehead	40.3	38.0
15 cm Rt. Parietal	40.3	31.2
15 cm Lt, Parietal	40.3	35.2
30 cm Vertex	61.0	76.9
30 cm Occiput	61.0	71.6
30 cm Forehead	61.0	60.7
30 cm Rt. Parietal	61.0	52.3
30 cm Lt, Parietal	61.0	53.9

Table 15
Effect of idealizing 11-month head as a sphere

Direction	Est. Accel, g	Neonate, avg. accel, g
15 cm Vertex	62.8	39.6
15 cm Occiput	62.8	69.4
15 cm Forehead	62.8	62.1
15 cm Rt. Parietal	62.8	49.4
15 cm Lt, Parietal	62.8	63.1
30 cm Vertex	96.2	67.6
30 cm Occiput	96.2	104.8
30 cm Forehead	96.2	109.9
30 cm Rt. Parietal	96.2	82.3
30 cm Lt. Parietal	96.2	74.3

DISCUSSION

This work provides an unusual view of the material properties of the pediatric head. Unusual in the sense that average Young's modulus estimated from experimental data for neonates is approximately 0.5 MPa. It rises to approximately 5 MPa at age 11-months as shown in Figure 10. This is in stark contrast with Young's moduli of various structural elements that together make up the head l such as scalp, skull, suture and membranes in FE models. FE models of pediatric heads have defined the modulus of these parts to be 17 MPa [scalp], 500 – 1500 MPa [skull]. 8 MPa [sutures] and 32 MPa [membranes]. The modulus estimated by this model is several orders of magnitude lower than those used in current FE models.
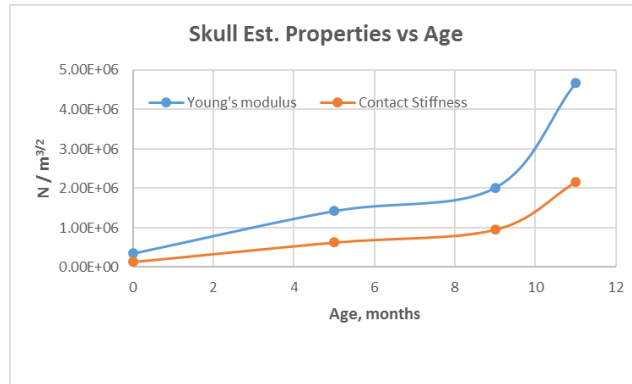
Figure 10: Contact stiffness and Young's modulus vs pediatric age.

My first inclination was to assume that the reasonableness of estimates with such a simple model was serendipitous. However, the model returns very reasonable estimates for all impact response variables including peak head deformation even when the model is extrapolated to estimate results of a 6.3 m/s impact. It also seems to return reasonable estimates when the head impacts foam whose modulus is several orders of magnitude lower than that of Steel.

Melvin (1995) might provide one explanation for the low value of modulus. Based on work by McPherson and Kriewall (1980), Melvin estimated that bone modulus scale factor for newborns as compared to adults was 0.243. Unpublished work by the author at GESAC, Inc established that Hybrid – III male 50th percentile head flesh had a Shore A hardness of 47 equivalent to 2.9 MPa. Based on Melvin's estimates, newborn modulus can be calculated to be 0.7 MPa and was recommended for the CRABI-6month dummy. In this study, modulus for neonates and 5-month old have been estimated to be 0.464 MPa and 1.43 MPa respectively. Therefore, Melvin's estimate fits well within the estimated developed in this study for the neonate – 6-month old age group. However, it would be very nice indeed to be able to estimate the composite modulus and structural stiffness of the neonate head directly from literature values of components.

Figure 10 shows estimated Contact Stiffness [CS] plotted against pediatric age. Data points were connected by a straight line. This is in keeping with Melvin, 1995 who observed that skull mechanical properties vary linearly with age from birth to 4.5 years. In any case, it is probably reasonable to assume that CS and modulus vary linearly in the small age range plotted in the figure.

Contact Stiffness [CS] is plotted in addition to estimated Young's modulus as CS includes Young's modulus and radius, a measure of child head geometry. Thus, CS can be different for the same Young's modulus depending on radius of the head.

Robustness and utility of a model is judged by its ability to provide interpolated and extrapolated estimates. Results of this study indicate that the model yields reasonable estimates for range of contact velocities from quasi-static compression to 6.3 m/s. Model estimates of peak acceleration when the head impacts softer surfaces are comparable to estimates of FE models in literature. So, it is possible to conclude that the Hertz contact model provides a robust, utile simulation tool for pediatric head impacts.

It has been noted previously in the RESULTS Section that model estimates away from the mean tend to differ significantly from experimental data. This is caused by the model producing single point estimates around mean experimental values. However, the model can be exercised to generate a range of estimates which will cover the extremes of experimental data. For example, the model currently uses mean pulse width to generate a single value of Young's modulus. The model can be used to generate a range of Young's moduli by using mean test pulse width ± SD as input. If these moduli are used, the model will provide good estimates of impact response variables. For example, if the E_{mean} + SD is used to calculate response in 15 cm drop tests for P03M head, the model returns acceleration of 47g as opposed to 38g listed in Table 5. Obviously, this new estimate quite close to the experimental

value of 49g. This procedure does not cure all problems with estimated values of variables. Analysts can use this procedure to generate a range of impact response variable values thus increasing the confidence in estimates.

CONCLUSION

- A method has been developed to estimate Young's modulus and Contact Stiffness of pediatric heads in conjunction with Hertz Contact model for head – rigid surface impacts.

- Study results indicate that the methodology and Hertz contact model yield reasonable estimates of head impact response variables.

- Proposed Model building methodology seems to yield reasonable estimates for a reasonably large pediatric age group ranging from <1-month to 11-month old. However, there is only 1 test specimen each for ages 5-, 8- and 11-months.

- There is a nearly 10-fold increase in the estimated modulus of the head in this age group.

- The reasonableness of impact response variables under various loading conditions indicates that Hertz model is appropriate to simulate pediatric head impact and is quite robust.

- This study indicates that modulus of the head to be much lower than the moduli of the components that constitute the pediatric head.

- The model seems capable of returning reasonable estimates for impact response variables for impacts against softer surfaces such as hard foam. The modulus of foam is several orders of magnitude lower than modulus of Steel and Aluminum which were used in infant cadaver and dummy head drop tests.

- Simulation results suggest that infant dummy heads can be molded out of 2-part elastomers which will lead to a head which is deformable, and very importantly has human-like moments of inertia about all axes. Such heads are also likely to be easy and inexpensive to manufacture and can be used to design head protection gear and vehicle components that might contact pediatric heads.

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APPENDIX

Pulse width in Hertz contact model for a sphere [Head] – Half space [Flat Plate] impact is given by (Abrate, 2001):

$$\tau = 3.2145 * \frac{M^{0.4}}{k^{0.4} * V^{0.2}} \qquad\qquad \text{(Equation A- 1)}$$

Where:

$\tau = Pulse\ width$

$V = Contact\ velocity$

$k = Contact\ stiffness = \frac{4}{3}\ E_* * R_h^{0.5} \qquad\qquad \text{(Equation A-2)}$

Where

$\frac{1}{E_*} = Contact\ Young's\ modulus\ of\ the\ plate\ and\ head = \frac{1-\nu_p^2}{E_p} + \frac{1-\nu_h^2}{E_h}$ (Equation A-3)

And

$$\frac{1}{R} = \frac{1}{R_p} + \frac{1}{R_h} \qquad\qquad \text{(Equation A-4)}$$

$R = Radius$ and subscripts "p" and "h" refer to rigid plate and head respectively

Procedure for calculating Young's modulus of Infant head

The procedure starts by recognizing that both infant head and dummy drop pulse widths are almost invariable with drop height [Rangarajan, et al, 2017(a, b)]. This information and known Young's modulus, mass and head radius of Aprica 2.5 dummy, and average mass and average radius of infant heads can be used to calculate Young's modulus of the infant head for the same contact velocity using Eq. A-1.

Thus, for the same contact velocity,

$$\frac{\tau_i}{\tau_d} = \frac{\frac{M_i^{0.4}}{k_i^{0.4}}}{\frac{M_d^{0.4}}{k_d^{0.4}}} \qquad\qquad \text{(Equation A-5)}$$

Or

$$\frac{\tau_i}{\tau_d} = \left[\frac{M_i * k_d}{M_d * k_i}\right]^{0.4} \qquad \text{(Equation A-5a)}$$

Subscripts "I" and "d" in Equation 2 and 2a refer to infant and Aprica 2.5 dummy respectively. Loyd, 2011 30 cm drop tests provide an estimated average infant head drop test pulse width of16.38 ms. Pulse width of the Aprica dummy 0.376 m drop tests is 8.6 ms. Thus, the of pulse widths ratio in Equation 2a above is 16.38 / 8.6 = 1.90

Table A.1 below lists values of variables in Equation 5a.

Table A- 1
Value of variables

Variable Name	Value	Units
Average infant head mass M_i	0.534	kg
Dummy head mass M_d	0.85	kg
Average infant head circumference	0.0323	m
Dummy head circumference	0.0340	m
Average infant head radius R_i	0.0514	m
Dummy head radius R_d	0.0542	m
Young's modulus infant E_i	TBD	MPa
Young's modulus Dummy E_d	3.42	MPa
Poisson's ratio infant head v_i	0.46	
Poisson's ratio dummy head v_d	0.46	

Data in Table A.1 can be used together with Equations A-1 through A-5a, to calculate Young's modulus for infant head. The calculated value is $4.64 * 10^5$ Pa. This value of Young's modulus is roughly equivalent to Shore A hardness of 13 and can be used in the Hertz contact model to estimate impact response variables. Estimated values of these variables are compared with experimental values for infant heads in the main text of this paper